Physics 212 – Statistical Mechanics

Introduction

Physics 212 is an advanced course in Statistical Mechanics. I will assume that you are familiar with the basis of Statistical Mechanics – the notion of entropy, the derivation of thermodynamics from the statistical point of view, the properties of classical and quantum ideal gases. In this course, we will go right to the most subtle and beautiful aspects of the subject – the theory of phase transitions between thermal states and the special properties of critical points, that is, points at which the transition between phases is continuous.

The essence of Statistical Mechanics is the idea of "coarse-graining". We do not try to keep track of the positions and momenta (in quantum mechanics, the wavefunctions) of each particle in the system. Rather, we acknowledge that particles can move and exchange momenta while maintaining a state that, to macroscopic observers, is constant and unchanging. We call this a "thermodynamic state", and we compute its properties (as well as we can) by integrating over the possible microscopic states that it contains. For simple systems, such a ideal gases and perfect crystals, we can carry out this integration explicitly. These models approximately describe real-world systems, and we can add interactions as perturbations.

These ideal statistical systems cannot have multiple thermodynamic phases. They have the property that any microstate of the system can readily make a transition to any other microstate, so the description of the thermodynamic state includes an integral over all of the microstates according to their statistical weight. Also, we will see that a finite physical system cannot have multiple phases. However, in an infinite system, microstates can form groups that are inaccessible to one another, and these can have distinct physical behavior. In practice, macroscopic systems, or even systems of micron size, can be large enough to show this behavior. So multiple phases, and transitions between them, are a complication of dealing with systems with very large numbers of degrees of freedom. One of the major issues in physics, which is still mysterious in many ways after a century of study, is the manner in which systems with many degrees of freedom can differ qualitatively from simple textbook system. The study of thermodynamic phases and phase transitions gives us an entryway into this challenging subject.

There is a second distinction that we can make among systems that show multiple phases. Typically, within each thermodynamic phase, the physics that determines the thermodynamic state is local. That is, the phase has an average quality, with deviations from this, and correlations among them, that are short-ranged. However, it is possible for a thermodynamic state to exhibit long-ranged correlations. Then the large-scale properties are determined by the collective action of a large number of degrees of freedom. This is true, in particular, at a continuous, or second-order, phase transition. In most cases, such a transition is associated with a scale-invariant thermodynamic state with correlations and important interactions on all length scales. There is a beautiful theory of this scale-invariant behavior that will be a major topic of this course.

The outline of the course is as follows:

Part 1 of the course will study idealized Hamiltonians describing statistical mechanical systems with second-order phase transitions. Much of our attention will be given to the simplest such system, the Ising model of a ferromagnet. I will present different methodsfor analyzing this model mathematically, which give different and complementary information about its thermodynamic phases. I will show ou that the presence of multiple thermodynamic phases can be proven to be a property of these simple models. But I will also try to persuade you that the presence of magnetism, and order-disorder phase transitions more generally, is not so obvious and requires answering some difficult questions.

Part 2 of the course will be devoted to Landau's phenomenological theory of order-disorder transitions. Landau analyzed systems of interest that have order-disorder transitions and tried to find a simple, unifying description. He emphasized especially the role of symmetry, which can be manifest in an disordered phase but is apparently broken in an ordered phase. As we follow his descriptions, we will gather the materials needed to describe the important, intrinsically quantum mechanical phase transitions to superfluidity and superconductivity.

Part 3 of the course will present the theory of thermal fluctuations at a second-order phase transition. Just at the transition point, the thermodynamic system has important thermal fluctuations on all length scales. I will analyze this system of scale-invariant fluctuations using a method introduced by Kenneth Wilson called (for no useful reason) the "renormalization group". This method provides a useful approach to other systems dominated by complex nonlinear interactions.

There will also be a Part 4 with some extra mathematical topics. We will see in Part 3 that systems with continuous symmetry in 2 dimensions give a special case that requires dedicated analysis. I will explain how this case works out. Also, the description of scale-invariant systems was much improved in the 1980's and even more in the 2000's using a symmetry called "conformal invariance". I will describe this and explain how it gives more precise answers to the questions studied in Part 3. This material will not be required, but it brings the study of critical phenomena to a fascinating conclusion.

The concepts that we will discuss in this course are some of the high points of 20th century physics. They are relevant to condensed-matter systems but equally well to nuclear and particle physics. The such famously mysterious concepts as, “topological solitons”, “Goldstone bosons”, the “Higgs mechanism”, and the “renormalization group” itself’ will appear in the course of our study. I hope that you enjoy the trip.

There are many well-written books on this subject. My favorite textbook of Statistical Mechanics is that of James Sethna, *Statistical Mechanics: Entropy, Order Parameters, and Complexity*. Sethna’s course is exceptionally well designed, but it is not quite at the level of this more advanced course. An alternative book with strong overlap with the material I will present is *Statistical Physics of Fields*, by Mehran Kardar. Personally, I find this book a little too technical for the typical students in this course, though theorists will find it a valuable reference. A new big book by Eduardo Fradkin titled “Quantum Field Theory” includes this material, which is, indeed, closely related to Quantum Field Theory. Finally, I highly recommend the volume *Statistical Physics* in the Landau and Lifshitz series. There is literally insight in every paragraph; even the footnotes prove important theorems to which other authors devote whole papers. Some additional references can be found on the course web page: `www.slac.stanford.edu/~mpeskin/Physics212/` .