Physics 212 – Quiz #3

(issued Thursday, November 12)

I will use the three quizzes in this course to assign grades, keeping in mind that anyone who hands in all of the problem sets is assured a reasonable grade in the course. Because these quizzes will be graded, please abide by these rules:

- The quizzes are open-book. You may use any reference resources that you find. However, please do not collaborate with other students or ask help from other people—except that, if you have any question about the quiz, please feel free to email me (mpeskin@slac.stanford.edu).
- The quizzes are posted at the course web site:

  `https://www.slac.stanford.edu/~mpeskin/Physics212/`

  Please hand in your solution (upload to Gradescope) within 24 hours of the time that you turn the page and begin to solve the quiz.
- Please write on your solution: "I acknowledge the Stanford Honor Code." and sign it.

Each quiz will be worth 25 points. Partial credit will be given.

1. Consider the Landau free energy

$$\beta G[S_1, S_2] = \int d^d x \left\{ \frac{1}{2}[(\vec{\nabla} S_1)^2 + (\vec{\nabla} S_2)^2] + \frac{1}{2} m^2 (S_1^2 + S_2^2) + \frac{b}{4}(S_1^4 + S_2^4) + \frac{c}{2} S_1^2 S_2^2 \right\} \quad (1)$$

Examine its behavior under the renormalization group near 4 dimensions. Notice that this theory becomes $SO(2)$-symmetric if we set $c = b$.

(a) Show that both the b term and the c term are marginal operators at the Gaussian fixed point in $d = 4$.

(b) Using methods from class, work out the RG equations for b and c in $d = 4$ to leading order, ignoring the effect of the m^2 term. Check your work by verifying that setting $c = 0$ gives the RG equation for an $n = 1$ Landau theory and that setting $c = b$ gives the RG equation for an $n = 2$ Landau theory.

(c) Write the RG equations for b and c in $d = 4 - \epsilon$.

(d) The equations in part (c) have several fixed points. Find them. At the minimum, there are fixed points with $c = 0$ and $c = b$ that reproduce the fixed points discussed in class.

(e) Sketch the RG flow directions in the (b, c) plane. Which fixed point is the most stable?

(f) Notice that, for some initial conditions, the RG flows carry the system to negative values of b. What is the physical interpretation of this?