Physics 212 – Quiz #2

(issued Monday, November 15; due Friday, November 19)

I will use the quizzes and the final in this course to assign grades, keeping in mind that anyone who hands in all of the problem sets is assured a reasonable grade in the course. Because these quizzes will be graded, please abide by these rules:

- The quizzes are open-book. You may use any reference resources that you find. You may use your mathematical software such as MatLab or Mathematica. However, please do not collaborate with other students or ask help from other people—except that, if you have any questions about the quiz, please feel free to email me (mpeskin@slac.stanford.edu).

- The quizzes are posted at the course web site:

  `https://www.slac.stanford.edu/~mpeskin/Physics212/`

  This is the cover page. Please hand in your solution (upload to Gradescope) within 24 hours of the time that you turn the page and begin to solve the quiz.

- Please write on your solution: "I acknowledge the Stanford Honor Code." and sign it.

- The quizzes should be turned by Friday of the week in which they are issued. If this is a problem for you, please email me.

Each quiz will be worth 25 points. Partial credit will be given.

1. Consider an XY spin model in 3 dimensions for which the crystal structure breaks the rotational symmetry to the group of 120° rotations, Z_3. Describe this model by a Landau theory, using the variable $\vec{m}(x) = (m_1(x), m_2(x))$, or, better, the complex variable $m(x) = m_1(x) + im_2(x)$. Consider the model at a temperature well below T_c. Take the breaking of rotational symmetry to be small, that is, consider the term that breaks rotational symmetry to have a small coefficient c. Notice that this model does not have $m(x) \to -m(x)$ symmetry, except in the limit $c \to 0$.

   (a) Give arguments that the Landau theory describing this model is

   $$G[m] = \int d^3x \left\{ \frac{1}{2}|\vec{\nabla} m|^2 - A|m|^2 - \frac{1}{2}c(m^3 + (m^*)^3) + \frac{B}{4}|m|^4 \right\} , \qquad (1)$$

   with $A, B > 0$. Notice that $G[m]$ must be real-valued; it is just a matter of simplifying the representation that $m(x)$ is a complex number. In the rest of this problem, treat c as a small parameter and work in the limit $c \ll A, B$, to the leading order in c only.

   (b) Find the three equilibrium states of this model for $c > 0$.

   (c) Find the three equilibrium states of this model for $c < 0$.

   (d) Show that, as $c \to 0$, a correlation length in this model becomes long. Find the behavior of this correlation length as $c \to 0$ from above.

   In the rest of this problem, assume that $c > 0$.

   (e) Show that there is a topologically stable domain wall linking two of the equilibrium states. Using dimensional analysis, find the thickness of the domain wall and find the surface tension of the domain wall, in each case, as a function of A, B, c, up to an overall constant.

   (f) Write a differential equation for the form of the domain wall.

   (g) Solve this equation and write an explicit formula for the shape of the domain wall.

   (h) Let the three equilibrium states of the model be called X, Y, and Z. It is possible to prepare the system in such a way that the left-hand side of the sample will be in the state X, the right-hand side of the sample will be in the state Z, and the middle of the sample wil be in the state Y. Is this a stable configuration? If the two domain walls approach one another, what will happen?