Physics 212 – Quiz #2

(issued Friday, October 30)

I will use the three quizzes in this course to assign grades, keeping in mind that anyone who hands in all of the problem sets is assured a reasonable grade in the course. Because these quizzes will be graded, please abide by these rules:

- The quizzes are open-book. You may use any reference resources that you find. However, please do not collaborate with other students or ask help from other people—except that, if you have any question about the quiz, please feel free to email me (mpeskin@slac.stanford.edu).
- The quizzes are posted at the course web site:

  `https://www.slac.stanford.edu/~mpeskin/Physics212/`

  Please hand in your solution (upload to Gradescope) within 24 hours of the time that you turn the page and begin to solve the quiz.
- Please write on your solution: "I acknowledge the Stanford Honor Code." and sign it.

Each quiz will be worth 25 points. Partial credit will be given.

1. Consider a fluid containing long molecules with different chemical groups on the heads and tails. At low temperature, this system can form a liquid crystal, with an order parameter $\vec{n}(\vec{x})$ that gives the local orientation of the molecules. The direction of $\vec{n}$ is from tail to head. In this system, unlike in "nematic liquid crystals", $\vec{n}$ and $-\vec{n}$ are distinct orientations.

   This system can be described by a Landau theory

$$G[n] = \int d^3x \left\{ \frac{1}{2} \sum_{i,k} (\nabla^i n^k)^2 + \frac{1}{2} a(T - T_c) \sum_k (n^k)^2 + \frac{b}{4} [\sum_k (n^k)^2]^2 \right\} \tag{1}$$

   where the indices i and k are summed over. Note that this is not the most general set of allowed terms; the additional terms are omitted for simplicity in the exam.

   Consider this fluid as a finite system confined between two parallel walls with separation R in the $\hat{x}$ direction. The planes are coated in such a way that they attract the heads of the molecules, so that at the walls the molecules point outward, with $|\vec{n}| = N_0$, where $N_0 > (aT_c/b)^{1/2}$. Also, assume that $R \gg (aT_c)^{-1/2}$.

   (a) Consider first the situation at $T \gg T_c$. Here we can ignore the bn^4 term in the free energy. Using this approximation, find $\vec{n}(x)$ satisfying the boundary conditions.

   (b) Use the mechanical analogy described in class to describe this solution. Then, describe qualitatively the form of the solution with $b \neq 0$. Describe how this solution, in which $\vec{n}(x)$ always remains parallel to $\hat{x}$, evolves as T is decreased to temperatures below T_c.

   (c) Estimate the free energy of the solution described in part (b), for which $\vec{n}(x) \parallel \hat{x}$, at temperatures much less than T_c.

   (d) Now consider solutions for $T < T_c$ in which the direction $\vec{n}$ is allowed to change. For temperatures much less than T_c, describe the optimal solution and estimate its free energy.

   (e) Using the expressions from (c) and (d), estimate the temperature at which the system crosses over from the first type of solution to the second. In particular, is the temperature of this crossover greater than or less than T_c?