

Physics 212 – Quiz #1

(issued Monday, October 31; due Friday, November 4)

I will use the quizzes and the final in this course to assign grades, keeping in mind that anyone who hands in all of the problem sets is assured a reasonable grade in the course. Because these quizzes will be graded, please abide by these rules:

- The quizzes are open-book. You may use any reference resources that you find. You may use your mathematical software such as MatLab or Mathematica. However, please do not collaborate with other students or ask help from other people—except that, if you have any questions about the quiz, please feel free to email me (mpeskin@slac.stanford.edu).
- The quizzes are posted at the course web site:

<https://www.slac.stanford.edu/~mpeskin/Physics212/>

This is the cover page. Please hand in your solution (upload to Gradescope) within 24 hours of the time that you turn the page and begin to solve the quiz.

- Please write on your solution: “I acknowledge the Stanford Honor Code.” and sign it.
- The quizzes should be turned by Friday of the week in which they are issued. If this is a problem for you, please email me.

There will be 2 quizzes in all. Each quiz will be worth 25 points. Partial credit will be given. The final will be worth 50 points.

1. Let's study the "clock model" with the Hamiltonian

$$\mathcal{H} = -J \sum_{i,\nu} \vec{\sigma}_i \cdot \vec{\sigma}_{i+\nu} \quad (1)$$

where $\vec{\sigma}_i$ is a unit vector from the set

$$\left\{ 1+ = (1, 0), \quad 1- = (-1, 0), \quad 2+ = (0, 1), \quad 2- = (0, -1) \right\}. \quad (2)$$

Consider this model on a d -dimensional (hyper)-cubic lattice.

- (a) Let p_{1+} , p_{1-} , p_{2+} , p_{2-} be the probability of finding each of the four vectors in (2) on a given site. These should satisfy

$$p_{1+} + p_{1-} + p_{2+} + p_{2-} = 1. \quad (3)$$

Compute the expected value of the terms in the Hamiltonian containing the site i if the spin at this site is $1+$, given these probabilities for the spins on the neighboring sites. Repeat for the other three possible spin orientations at i .

- (b) Using these results, construct the mean field self-consistency equations for the four probabilities. These equations should automatically respect eq. (3).
- (c) One possible pattern of symmetry breaking is a magnetization in the $\hat{1}$ direction. To test for this, make the ansatz

$$p_{2+} = p_{2-} \quad (4)$$

Show that the mean field equations reduce to an equation for

$$p_1 = p_{1+} - p_{1-} \quad (5)$$

(It is convenient also to define $p_2 = p_{2+} - p_{2-}$; this is 0 if eq. (4) holds.) Show that there is an order-disorder transition at a finite temperature T_c , and compute T_c .

- (d) Another pattern of symmetry breaking is a magnetization in the $(\hat{1} + \hat{2})/\sqrt{2}$ direction. Test for this with the ansatz

$$p_{1+} = p_{2+}, \quad p_{1-} = p_{2-}. \quad (6)$$

Show that an order-disorder transition is predicted along this direction, and with the same critical temperature T_c .

- (e) For each of these cases, compute the expectation value of the magnetization just below T_c . Show that the magnetization in case (d) is less than the magnetization in case (c).

This leads to the fact that the potential difference between the symmetric and broken-symmetry states is smaller in case (d), and therefore case (c) is preferred. (You don't have to prove this.) Assume in the next part that the symmetry-breaking pattern just below T_c has $p_1 > 0$, $p_2 = 0$.

- (f) The state with $p_1 > 0$, $p_2 = 0$ breaks the reflection symmetry $1+ \leftrightarrow 1-$, but it preserves the reflection symmetry $2+ \leftrightarrow 2-$. In principle, it is possible that, at a lower temperature, the second reflection symmetry is also broken, leading to a nonzero p_2 . By analyzing the mean-field equations for $T < T_c$, show that this never happens.