

Physics 212 – Quiz #1  
(issued Thursday, October 1)

I will use the three quizzes in this course to assign grades, keeping in mind that anyone who hands in all of the problem sets is assured a reasonable grade in the course. Because these quizzes will be graded, please abide by these rules:

- The quizzes are open-book. You may use any reference resources that you find. However, please do not collaborate with other students or ask help from other people—except that, if you have any question about the quiz, please feel free to email me (mpeskin@slac.stanford.edu).
- The quizzes are posted at the course web site:

<https://www.slac.stanford.edu/~mpeskin/Physics212/>

Please hand in your solution (upload to Gradescope) within 24 hours of the time that you turn the page and begin to solve the quiz.

- Please write on your solution: “I acknowledge the Stanford Honor Code.” and sign it.

Each quiz will be worth 25 points. Partial credit will be given.

1. Consider an Ising magnet on a square lattice in 2 dimensions, with the Hamiltonian

$$\mathcal{H} = - \sum_{i\nu} JS_i S_{i+\nu} - \sum_i H\mu S_i . \quad (1)$$

and periodic boundary conditions. The magnetization of the model is

$$M = \mu \sum_i \langle S_i \rangle \quad (2)$$

(a) Prove that, in the high-temperature phase, the magnetic susceptibility at zero external field  $H$  is given by

$$\chi(T) = \left. \frac{\partial M}{\partial H} \right|_T = N\mu^2\beta \sum_j \langle S_j S_0 \rangle , \quad (3)$$

where  $N$  is the total number of sites and 0 denotes one specific site on the lattice.

(b) Work out a series expansion for  $\chi(T)$  in powers of  $z = \tanh \beta J$ . Compute all terms up to and including order  $z^5$ .

(c) Show that, in the expansion

$$\frac{A}{(z_* - z)^\gamma} = \frac{A}{z_*} (1 + a_1 z + a_2 z^2 + \dots) \quad (4)$$

the ratio of coefficients is

$$a_n/a_{n-1} = \left( \frac{\gamma + n - 1}{n} \right) \frac{1}{z_*} . \quad (5)$$

(d) Using this observation and your results for the series, estimate  $z_*$  and  $T_c$ . Compare to the mean field theory result  $T_c = 4J$  and to the exact result  $T_c = 2.27J$ .