Physics 212 – Problem Set # 7

(due Thursday, May 20)

1. Sethna, problem 9.4.

   Yes, I know that I gave you the explicit solution for the domain wall configuration in the lecture notes. But Sethna discusses the scaling properties of the solution, the estimation of its energetics and size, and a classical mechanics analogy that is useful in obtaining the solution.

2. Consider a magnet in which the spins are invariant with respect to the symmetry group of a cubic crystal lattice. Let us write the Landau effective free energy in terms of an order parameter M^i, $i = 1, 2, 3$. In this case, the most general Landau free energy has one additional term beyond those considered in class:

$$G(M) = \frac{1}{2}a(T - T_c)(M^i)^2 + \frac{1}{4}b((M^i)^2)^2 + \frac{1}{4}c(M^i)^4 , \tag{1}$$

   with sums over repeated indices. The first two terms are invariant under the full group of three-dimensional rotations acting on M^i, but the last term has a lower degree of symmetry.

   (a) Find the ground states of the system for $T < T_c$. Show that, for $c < 0$, the ground states of the system are

$$\left\langle \vec{M} \right\rangle = (M_0, 0, 0), \quad (0, M_0, 0), \quad (0, 0, M_0)$$

   for an appropriate M_0. Where are the ground states in the case $c > 0$?

   (b) Consider the case $c < 0$ in more detail. Expand $G(M)$ about the ground state $(M_0, 0, 0)$. Compute the spin-spin correlation function $\langle s^i(x)s^j(0)\rangle$ using the approximate method discussed in class. Show that there are two different correlation lengths, one for the fluctuations of s^1, the other for the fluctuations of s^2, s^3. What happens when $c \to 0$? Why?

   (c) Compute the spin-spin correlation function in one of the ground states for $c > 0$.

   (d) Compute the magnetic susceptibility at zero field

$$\chi^{ij} = \partial M^i / \partial h^j |_{h=0}$$

   for $T > T_c$ and for the three cases with $T < T_c$: (1) $c = 0$, (2) $c < 0$, (3) $c > 0$. In general, the magnetic susceptibility will be a tensor. How are the anisotropies of χ related to the anisotropies of the spin-spin correlation function?

3. How much current can flow through a superconducting wire without destroying the superconductivity? Start from the Ginzburg-Landau free energy:

$$\int d^3x \left\{ \frac{1}{2m_*} \left| -i\hbar\vec{\nabla}\Phi - \frac{e_*}{c}\vec{A}\Phi \right|^2 + \frac{1}{2}a(T-T_c)|\Phi|^2 + \frac{1}{4}b|\Phi|^4 \right\} \tag{2}$$

For a thin wire, we can look for a solution where $\vec{A} = 0$ in the wire and the order parameter is uniform across the wire cross section:

$$\Phi(z) = \hat{\Phi} \cdot e^{ikz} \tag{3}$$

The velocity of the superflow is $v_s = \hbar k/m_*$. Find the value of $\hat{\Phi}$ that minimizes the energy for fixed v_s. Then maximize the electric current density

$$j = e_*|\Phi|^2 v_s \tag{4}$$

with respect to v_s to find the maximum allowed current. How does the maximum surface magnetic field in this geometry compare to the critical field that destroys superconductivity in a large sample?