Physics 212 – Problem Set # 4

(due Thursday, April 29)

1. Sethna, problem 6.8.
2. Sethna, problem 6.11.
3. Sethna, problem 6.12.
4. The *density-density correlation function* of a fluid is defined by

$$\rho(\vec{x}_1, \vec{x}_2) = \left\langle \sum_{i \neq j} \delta(\vec{x}_i - \vec{x}_1)\delta(\vec{x}_j - \vec{x}_2) \right\rangle \tag{1}$$

Analyze this in the limit of large volume, in particular, ignoring any effects from atoms being near the walls of the container.

(a) Show that, for an ideal gas, $\rho(\vec{x}_1, \vec{x}_2) = n^2$.

(b) Argue that, for spherical atoms interacting through central forces, $\rho(\vec{x}_1, \vec{x}_2)$ should depend only on $r = |\vec{x}_1 - \vec{x}_2|$. Assume this in the following.

(c) Compute ρ to lowest order in the density. Show that

$$\rho(r) = n^2 \cdot e^{-\beta V(r)}$$

Sketch this result for a Lennard-Jones potential. Is it sensible?

(d) Show that ρ can be computed from the grand canonical ensemble using the diagrams we introduced for the Mayer cluster expansion. Work out the prescription for evaluating diagrams. In your final result, no factors of the volume should appear.

(e) Compute ρ to order n^3 for hard spheres: $V(r) = 0$ for $r > a$, ∞ for $r < a$.