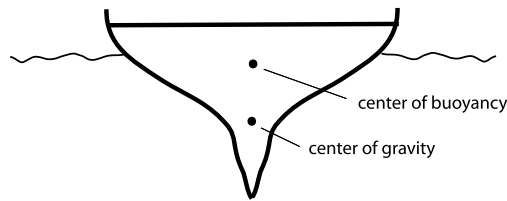


Physics 211 – Problem Set # 1

(due Thursday, January 14)

Many of the problems for this course will be taken from the textbook of Blandford and Thorne, *Applications of Classical Physics*. Problems from this source will be written out in full but will be referenced, *e.g.*, ‘(B&T 12.6)’.

1. Use Archimedes’ Law to explain qualitatively the conditions under which a boat floating in still water will be stable to small rolling motion from side to side. The standard approach to this problem is to introduce an appropriately defined *center of buoyancy* inside the boat, as in the figure. You may assume that the center of buoyancy keeps its position relative to the boat as the boat heels over and derive the criterion with that assumption. (Actually, the assumption is not quite correct for a boat whose hull does not have a circular cross section.) (B&T 12.4)



2. Work out the shape of a planet spinning about the \hat{z} axis with the angular velocity Ω . It is easiest to work in the rotating frame, where the rotation appears as a centrifugal force with potential $\Phi = -\frac{1}{2}\Omega^2(x^2 + y^2)$. Let R_p be the radius of the planet at the poles and R_e be the radius at the equator. (B&T 12.6)
 - (a) Consider first a planet for which most of the mass is concentrated at the center and the outer parts are a low-mass fluid. Show that, if g is the acceleration of gravity at the surface of the planet,

$$R_e - R_p \approx \frac{\Omega^2 R^2}{2g}$$

- (b) Show that, for a non-rotating planet of fixed mass M and uniform density ρ , the gravitational potential is

$$\Phi = \frac{2\pi G_N \rho r^2}{3}$$

- (c) Argue that, for a slowly spinning planet that is a uniform density fluid, the gravitational potential is modified to

$$\Phi = \frac{2\pi G_N \rho r^2}{3} + Ar^2 P_2(\mu)$$

where A is a constant, $P_2(x)$ is the Legendre polynomial, and μ is the sine of the latitude. What happened to $P_1(\mu)$?

- (d) Give an equivalent expansion for the potential outside the planet.
- (e) Match the two potentials by insisting the the gravitational force has no discontinuity across the boundary.
- (f) Now transform to the rotating frame, add the centrifugal potential, and show that

$$R_e - R_p \approx \frac{5\Omega^2 R^2}{4g}$$

where, again, g is the gravitational acceleration at the surface.