Physics 211

Continuum Mechanics

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OUTLINE:

- Basic equations of fluid dynamics
- Potential flow
- Viscosity
- Flows of viscous fluids
- Low Reynolds number
- Boundary layers
- Fluid instabilities
- Turbulence
- Wakes, boundary layers, separation
- Thermal conduction and convection
- Rotating fluids
- Compressible fluids
- Shock waves
- Magnetohydrodynamics

1 Introduction

This is a course on *continuum mechanics*, mainly covering topics in fluid mechanics. A fluid is a condensed matter state that is arbitrarily deformable, that is, a state in which there is no resistance to a deformation that is carried out sufficiently slowly. This criterion distinguishes *liquids* and *gases* from solids. It means that a primary feature of the dynamics is flow from one point to another.

This course follows a course in *particle mechanics*. The dominant spirit of the standard course in particle mechanics is the idea that particles interact through conservative forces. The constraint of energy conservation is highly restrictive. It allows elegant mathematical descriptions of the dynamics such as the Lagrangian and Hamiltonian formalisms.

Fluid mechanics has a very different spirit. The interesting aspects of fluid mechanics involve equations that *nonlinear* and *dissipative*. In this realm, we lose the general constraints of Hamiltonian mechanics. We need to gain insight by solving individual examples one by one. At the same time, these examples will have a wonderful geometrical character, so that we can obtain that insight by visualizing this geometry.

Particle mechanics is a standard topic in the physics curriculum. Fluid dynamics is less so. But fluids play an important role in the whole macroscopic world, at all scales from the those of cells and bacteria to those of stars and galaxies. For this reason also, fluid dynamics ought to be part of any physicist's education.

As a theorist, I am especially fascinated by the way that fluid mechanics illustrates the mathematics of nonlinear systems through geometry. In this course, we will focus much attention on the ways that the solutions of partial differential equations can behave and misbehave. I hope that the insights we gain will be useful to you for many problems in physics.

Here is an outline of the course material. I will begin by writing the *basic partial differential equations* that govern fluid motion. We will then discuss two idealized limits of fluid motion—*potential flow*, in which we ignore the viscosity (internal friction) of fluids and examine ideal, smooth flow patterns, and *highly viscous flow*, in which the flow patterns are smoothed by dissipation. We will then ask how a fluid behaves between these two extremes. In this study, we will find many sorts of misbehavior—*boundary layers*, in which apparently unimportant effects of viscosity come to dominate in small regions of the flow, and *instabilities*, in which a smooth flow is disrupted by perturbations. Eventually, following these routes, a fluid can become *turbulent*. I will discuss all of these concepts in the context of ideal incompressible fluids. Then, in the last part of the course, I will discuss the new effects that appear when we allow fluids various types of internal structure. These effects will include those induced by *heating and convection*, by *rotation*, by *compressibility*, and by interaction with *electric and magnetic fields.*

Much of this material will be drawn from the excellent textbooks that exist for this subject, especially

- Landau and Lifshitz, *Fluid Mechanics*
- Batchelor, *An Introduction to Fluid Dynamics*
- Blandford and Thorne, *Applications of Classical Physics*

Additional literature is linked at the bottom of the course Web page:

`http://www.slac.stanford.edu/~mpeskin/Physics211/`