Physics 210 – Problem Set # 7

(due Thursday, November 11)

1. Here are some further examples of canonical transformations of a 2-dimensional phase space.

   (a) Consider
   $$Q = \log(\frac{1}{q}\sin p) \ , \qquad P = q\cot p \ . \tag{1}$$
   Compute the Jacobian of this transformation and show that it is symplectic.

   (b) Consider
   $$Q = \log(1 + \sqrt{q}\cos p) \ , \qquad P = 2(1 + \sqrt{q}\cos p)\sqrt{q}\sin p \tag{2}$$
   Compute the Jacobian of this transformation and show that it is symplectic.

   (c) Show that the following generating function generates the transformation in part (b):
   $$F_3(Q, p) = -(e^Q - 1)^2 \tan p \tag{3}$$

   (d) Find a generating function which generates the transformation of part (a).

   [This is problems 9.4 and 9.6 from Goldstein's book.]

2. Consider the following sets of differential equations. Each depends on a real-valued parameter A. In each case, the behavior of the flow pattern in the phase plane changes qualitatively at a certain value of A. This sudden change is called a *bifurcation*.

   In the examples below, there is a bifurcation at $A = 0$ in examples (a) and (b). In (c), there is a bifurcation at a positive value of A. In each case, for systems on each side of the bifurcation point, find the equilibrium points and determine their stability analytically, and sketch the flow pattern in the phase plane.

   (a) $\dot{x} = y \ , \qquad \dot{y} = x - Ax^3$

   (b) $\dot{x} = xy \ , \qquad \dot{y} = -(x^2 - A) - y$

   (c) $\dot{x} = x - x^2 - Ay^2 \ , \qquad \dot{y} = y - y^2$

   Please feel free to solve these equations numerically to get an idea of of the flow pattern. However, you also carry out the stability analysis analytically.