Sept. 21

Introduction

The motto for this course is: "Physics is that subset of human experience that can be analyzed from the point of view of coupled harmonic oscillations." I do not think that this statement degrades physics. On the contrary, it is truly amazing what a wide range of phenomena arise when simple periodic systems are coupled together by nonlinear interaction.

In quantum theory, every particle is viewed as an excitation of a quantum oscillator, and every interaction is a nonlinear coupling whose effects can be explored in greater or less detail. As these interactions ramify, we see Nature in all its complexity.

To provide the foundation for that study, the first course in the graduate physics curriculum is a course in classical mechanics, emphasizing especially periodic systems and their perturbations. That course gives a foundation for sophisticated approaches to the dynamics of these systems, and to their quantization. My main purpose this term will be to provide you that foundation.

In this course, we will study the mechanics of simple systems of particles. There are three parts to this study. First, we must understand the most straightforward properties of these systems and the practical problems that arise when they are coupled nonlinearly. Second, we must learn how to visualize the dynamics of these systems so that their qualitative behavior can be found intuitively. Third, we must learn the various formalisms that have been developed to further the first two goals in increasingly sophisticated ways. That formalism has been created by many of the giants of mathematical physics—from Euler, Lagrange, Hamilton, and Jacobi to Poincaré and to Arnold and Kolmogorov in our own era – and it is a privilege to roam through the ideas that they have discovered.

In more detail, the course will cover the following items:

1. Elements of Newtonian particle mechanics: oscillations, planetary motion.

2. Lagrangian mechanics: calculus of variations, the connection between symmetries and conservation laws.

3. The rotation group: parametrization of rotations, and dynamics of rigid bodies.

4. Small oscillations: stability, perturbation theory, singular and secular perturbations.

5. Hamiltonian mechanics: phase space, canonical transformations, Hamilton-Jacobi theory, applications to perturbation theory.
6. Chaos: transition to chaos in simple and in Hamiltonian system, KAM theory.
7. Lagrangian formalism for fields, and realization of symmetries on fields.

There are many excellent textbooks on classical mechanics. The official textbook for this course is

Fetter and Walecka, Theoretical Mechanics of Particles and Fields

This is available in an inexpensive Dover edition. Dover has also published a supplement

Fetter and Walecka, Nonlinear Mechanics

from which I will also draw some material. Other highly recommended texts are

Goldstein, Classical Mechanics
(go to a used book store and get the 1st edition if possible)
Landau and Lifshitz, Mechanics
Arnold, Mathematical Methods of Classical Mechanics

Those of you who are comfortable with the language of tangent bundles, or would like to see how Hamiltonian mehanics is expressed in this language, should also explore

Abraham and Marsden, Foundations of Mechanics