Physics 152/252 – Problem Set # 9

(due Thursday, June 2)

1. The top quark is so heavy that it can decay to an on-shell W boson and a bottom quark. The decay matrix element is

$$\mathcal{M}(t \to bW^+) = \frac{g}{\sqrt{2}} u(b)_L^\dagger \overline{\sigma}^\mu u_L(t) \epsilon^{\mu *}(W) \tag{1}$$

Ignore the b quark mass. In this limit, the b is always left-handed. The spinor $u_L(t)$ is the top 2 components of the top quark Dirac spinor. In the following, work in the top quark rest frame, and assume for definiteness that the top quark is polarized in the $+\hat{3}$ direction.

   (a) Write formulae for the energies and momenta of the final state W and b in terms of m_t, m_W. Assuming that the W^+ momentum is at an angle θ with respect to the $\hat{3}$ axis,

$$p_W = (E_W, p_W \sin\theta, 0, p_W \cos\theta) \tag{2}$$

   write the b quark momentum vector and the b spinor $u_L(p_b)$.

   (b) Compute the partial width $\Gamma(t \to b_L W^+_-)$, to a W boson of helicity (-1). The appropriate W polarization vector is

$$\epsilon(W) = \frac{1}{\sqrt{2}}(0, \cos\theta, -i, -\sin\theta) \tag{3}$$

   (c) Compute the partial width $\Gamma(t \to b_L W^+_+)$, to a W with positive helicity. You should get zero for the result. Why is this process forbidden?

   (d) Show that the polarization vector $\epsilon(W) = (0, 0, 0, 1)$ boosts to

$$\epsilon(W) = (\frac{p_W}{m_W}, 0, 0, \frac{E_W}{m_W}) \tag{4}$$

   where the W is boosted along the $\hat{3}$ axis. This is the longitudinal polarization vector for a massive gauge boson.

   (e) Rotate this polarization vector appropriately, and compute the partial width $\Gamma(t \to b_L W^+_0)$, to a W with helicity zero.

   (f) Compute the total width of the top quark.

   (g) Compute the ratio of rates for top quark decays to transverse and longitudinal W bosons. Which mode is dominant?

(h) To understand the result of part (g), remember that the longitudinal polarization state of a W boson was originally a Higgs boson. The coupling of the top quark to a W boson has strength g, but the coupling of the top quark to a Higgs boson has strength y_t. Compute the ratio

$$y_t^2/g^2 \tag{5}$$

in terms of m_W and m_t. How does this compare to the result of part (g)?