Physics 152/252 – Problem Set # 8

(due Thursday, May 26)

1. This problem studies weak interaction decays of the τ lepton. The τ is a heavy lepton. The τ and its neutrino ν_τ couple to the weak interaction in the same way as the electron and the muon. The mass of the τ is 1777 MeV.

   (a) The V–A theory predicts that the τ will decay by $\tau^- \to \nu_\tau e^- \overline{\nu}_e$ and $\tau^- \to \nu_\tau \mu^- \overline{\nu}_\mu$. These processes are very similar to muon decay. Compute the partial widths for these decays, using formulae derived in class. (You may ignore the muon mass.)

   (b) Next consider the partial width for the τ to decay to quarks: $\tau \to \nu_\tau d\overline{u}$. Assume that the τ mass is large enough that we can ignore QCD and all quark masses. Then the calculation is just parallel to that for $\tau^- \to \nu_\tau e^- \overline{\nu}_e$. QCD color must be included. We saw in class that the first QCD correction is obtained by mutiplying the zeroth order result by
   $$(1 + \frac{\alpha_s(m_\tau)}{\pi}) \tag{1}$$
   Combining this factor into the zeroth order computation, compute the partial width for $\tau \to \nu_\tau d\overline{u}$.

   (c) From the results of parts (a) and (b), compute the tau lifetime and the branching ratio of the tau to leptonic modes. How do these numbers compare with the measured values reported by the Particle Data Group?

   (d) A specific hadronic decay of the τ is $\tau^- \to \nu_\tau \pi^-$. Work out the kinematics of this reaction in the frame where the τ is at rest. Let the τ have its spin parallel to the $\hat{3}$ axis, and let the π^- go off at an angle θ with respect to the $\hat{3}$ axis. Write the momentum vectors of the π^- and the ν_τ. Write the spinors $u(p)$ for the τ and the ν_τ. The 2-component spinor in $u_L(\nu_\tau)$ should be left-handed with respect to the ν_τ direction of motion.

   (e) Compute the matrix element for the decay $\tau^- \to \nu_\tau \pi^-$. The calculation is similar to that for π decay to $\mu\nu$. For the hadronic half of the amplitude, you will need the identity
   $$\left\langle \pi^-(p) \right| j_L^{-\mu} \left| \Omega \right\rangle = i f_\pi p^\mu / \sqrt{2} \tag{2}$$
   where j_L^- is the charge-changing weak interaction current. (This result is derived in the class notes.) For the leptonic half of the amplitude, use the explicit spinors for the τ and the ν_τ derived in (d).

   (f) Compute the partial width for $\tau^- \to \nu_\tau \pi^-$, using $f_\pi = 93$ MeV. Predict the branching fraction for this decay model, and compare to the Particle Data Group value.

(g) Work out the angular distribution of the pion in $\tau^- \to \nu_\tau \pi^-$ relative to the τ spin direction. Notice that the pion direction is correlated with the τ spin and allows this spin direction to be measured.

2. The Z can appear as an intermediate state in e^+e^- annihilation. The contributions from the intermediate virtual γ and Z should be added in the amplitude and can interfere.

   (a) Considering only an intermediate γ, recall the differential cross sections for $e^-_L e^+_R \to \mu^-_L \mu^+_R$, $e^-_L e^+_R \to \mu^-_R \mu^+_L$, $e^-_R e^+_L \to \mu^-_L \mu^+_R$, $e^-_R e^+_L \to \mu^-_R \mu^+_L$ computed in class. For example,
   $$\frac{d\sigma}{d\cos\theta}(e^-_L e^+_R \to \mu^-_R \mu^+_L) = \frac{\pi\alpha^2}{2s}(1+\cos\theta)^2 \ . \tag{3}$$

   (b) Draw the Feynman diagrams for virtual γ and virtual Z and compare them. Show that the cross section for $e^-_L e^+_R \to \mu^-_L \mu^+_R$ in the full electroweak theory is given by multiplying the result in (a) by
   $$\left| 1 + \frac{1}{c_w^2 s_w^2}(\frac{1}{2} - s_w^2)^2 \frac{s}{s - m_Z^2 + i m_Z \Gamma_Z} \right|^2 \tag{4}$$

   (c) In a similar way, compute the cross sections for the other possible helicity states.

   (d) The *forward-backward asymmetry* A_{FB} for the reaction $e^+e^- \to \mu^+\mu^-$ is defined by
   $$A_{FB} = \frac{\sigma(\cos\theta > 0) - \sigma(\cos\theta < 0)}{\sigma(\cos\theta > 0) + \sigma(\cos\theta < 0)} \ . \tag{5}$$
   Compute the forward-backward asymmetry for the polarized reaction in (a) and show that it equals 3/4.

   (e) Now consider the A_{FB} for the unpolarized process $e^+e^- \to \mu^+\mu^-$. A_{FB} obtains contributions from each of the 4 possible polarized reactions. Show that $A_{FB} = 0$ for $\sqrt{s} \ll m_Z^2$. Find A_{FB} just on the Z resonance, where the contribution from virtual γ can be ignored.

   (f) Write the leading term in the expression for the cross section in (b) in the limit $s \gg m_Z^2$.

   (g) Consider the unbroken $SU(2) \times U(1)$ theory. In this theory, the process $e^+e^- \to \mu^+\mu^-$ is mediated by virtual A^3 and B boson exchange. Compute the cross section for the polarized process in (b). You should find agreement with your answer in (e).

   (h) Check this agreement for the other three helicity states. Apparently, spontaneous symmetry breaking only affects cross sections at low energy. In some sense, a spontaneously broken symmetry is restored at sufficiently high energy.