

# Physics 152/252 – Problem Set # 6

(due Thursday, May 12)

1. The Drell-Yan process is the reaction in a  $pp$  or  $p\bar{p}$  collision that produces a muon or electron pair. The underlying process is  $q\bar{q} \rightarrow e^+e^-$  or  $\mu^+\mu^-$ . In this problem, we will work out the parton-model description of this process. Ignore all quark and lepton masses.

(a) Write the total cross section for  $q_f\bar{q}_f \rightarrow \mu^+\mu^-$  as a function of the quark-antiquark center of mass energy, ignoring all fermion masses. The factor for color should be  $\frac{1}{3}$  rather than the 3 in the formula for the  $e^+e^- \rightarrow q_f\bar{q}_f$  cross section. Why? Aside from this factor, you can get the rest of the expression from our analysis of  $e^+e^- \rightarrow q_f\bar{q}_f$ .

(b) In the parton model, the cross section for  $pp \rightarrow \mu^+\mu^- + X$  is given by

$$\sum_f \left[ \int dx_1 f_f(x_1) \int dx_2 f_{\bar{f}}(x_2) \sigma(q_f(x_1 P_1) \bar{q}_{\bar{f}}(x_2 P_2) \rightarrow \mu^+\mu^-) + (f \leftrightarrow \bar{f}) \right] \quad (1)$$

where the sum runs over quark flavors. Write an expression for  $\hat{s}$  for the parton reaction in terms of  $x_1$ ,  $x_2$  and  $s$  for the  $pp$  collision. Note that  $\hat{s} = M^2$ , the mass-squared of the observed  $\mu^+\mu^-$  system.

(c) Working in the  $pp$  CM system, write the 4-vectors of the initial quark and antiquark. Let  $(\mathcal{E}, 0, 0, \mathcal{P})$  be the sum of these momenta. This is also the total momentum of the  $\mu^+\mu^-$  system. The *rapidity*  $y$  of the  $\mu^+\mu^-$  system is defined by

$$\tanh y = \mathcal{P}/\mathcal{E} \quad (2)$$

Verify that  $\mathcal{E} = M \cosh y$  and  $\mathcal{P} = M \sinh y$ . Write an expression for  $y$  in terms of  $x_1$  and  $x_2$ .

(d) Write the converse expressions for  $x_1$  and  $x_2$  in terms of  $y$  and  $M$  (with  $s$  fixed). Notice that in this process, as in deep inelastic scattering, we can determine the values of the parton momentum fractions by measuring only the lepton momenta.

(e) In the cross section formula, change variables from  $x_1, x_2$  to  $M, y$ . Use the Jacobian determinant to convert  $dx_1 dx_2$  to  $dM dy$ . Write the formula for

$$\frac{d}{dM dy} \sigma(pp \rightarrow \mu^+\mu^- + X) \quad (3)$$

(f) The equation (1) has two terms, one with a quark from proton 1 and an antiquark from proton 2, and the other in which the antiquark comes from proton 1. We might break this down further into contributions from valence quarks and sea

quarks annihilating with antiquarks. For concreteness, think about the LHC, where the  $pp$  center of mass energy is 13000 GeV, a typical value of  $M$  is 90 GeV, valence quarks have  $x > 0.05$ , and sea quarks have  $x < 0.01$  (see Fig. 3.1 in the text). Argue that, near  $y = 0$ , most annihilations are sea with sea. At what value of  $y$  should sea with valence annihilations be important?

2. The nonlinear terms in the QCD Lagrangian lead to the following expression for the matrix element for a gluon to radiate a gluon:

$$\mathcal{M}(g^a(p, \epsilon_p) \rightarrow g^b(q, \epsilon_q) + g^c(k, \epsilon_k)) = g_s f^{abc} \left[ (k+p) \cdot \epsilon_q^* \epsilon_k^* \cdot \epsilon_p - (p+q) \cdot \epsilon_k^* \epsilon_p \cdot \epsilon_q^* + (q-k) \cdot \epsilon_p \cdot \epsilon_q^* \cdot \epsilon_k^* \right] \quad (4)$$

where  $f^{abc}$  is the structure constant in the commutator of two group generators. For  $SU(2)$ ,  $f^{abc} = \epsilon^{abc}$ .

- (a) To make the expression (4) more symmetric among  $p, q, k$ , replace the incoming momentum  $p$  by an outgoing momentum  $(-p)$  and  $\epsilon_p$  by  $\epsilon_p^*$ . Show that the resulting expression is completely symmetric under interchange of any two gluons, as required by Bose symmetry.
- (b) Set up the kinematics of almost collinear gluon radiation for  $g(p) \rightarrow g(q)g(k)$ , as described in the lecture for  $q(p) \rightarrow g(q)q(k)$ . To order  $q_T$ , the three 4-vectors are

$$p \approx (E, 0, 0, E) \quad q \approx (zE, q_T, 0, zE) \quad k \approx ((1-z)E, -q_T, 0, (1-z)E)$$

Modify the 3 component of  $q$  and  $k$ , as we did in class, so that these vectors satisfy  $q^2 = k^2 = 0$  to order  $q_T^2$ , and then modify the 3 component of  $p = q + k$ .

- (c) Write the polarization vectors (for L and R polarizations) for the three gluons. These expressions should be correct to order  $q_T^1$ . Make tables of the values of  $(\epsilon \cdot p)$  and  $(\epsilon \cdot \epsilon)$  needed to compute the matrix elements in (b) for all possible polarizations.
- (d) Work out the matrix element for  $g_R \rightarrow g_L g_L$ , to order  $q_T^1$ , and show that it is zero.
- (e) Work out the matrix element for  $g_R \rightarrow g_R g_L$ . And, changing just what needs to be changed, work out the matrix element for  $g_R \rightarrow g_L g_R$ .
- (f) Work out the matrix element for  $g_R \rightarrow g_R g_R$ .
- (g) Show that the matrix elements for  $g_L \rightarrow g g$  are given by reversing all polarizations in the previous calculations.
- (h) Now we must square the matrix elements, average over initial spins and colors, and sum over final state spins and colors. To do the sum over colors, you will need the identity

$$\sum_{cd} f^{acd} f^{bcd} = 3\delta^{ab} \quad (5)$$

for  $SU(3)$ . For  $SU(N)$ , this expression equals  $N$ . Show explicitly that this works for  $SU(2)$ :  $\epsilon^{acd} \epsilon^{bcd} = 2\delta^{ab}$ .

- (i) Square the matrix elements, sum over final spins and colors, and average over initial spins and colors. Compare this expression to the analogous sum over matrix elements for  $q \rightarrow gq$  derived in the lecture (eq. (3.30) with  $Q_f^2 e^2 \rightarrow \frac{4}{3} g_s^2$ ).
- (j) Finally, following the derivation for  $q \rightarrow gq$  and just changing what needs to be changed, derive the following expressions for the emission of an almost-collinear gluon from a gluon:

$$\text{Prob}(g \rightarrow gg) = \frac{3\alpha_s}{\pi} \int dz \int \frac{dq_T}{q_T} \frac{1 + z^4 + (1 - z)^4}{z(1 - z)}$$