Physics 152/252 – Problem Set # 5

(due Thursday, May 5)

1. Consider the deep inelastic scattering of polarized electrons on polarized protons. There are four independent different possible initial states: $e_L^- p_L$, $e_L^- p_R$, $e_R^- p_L$, and $e_R^- p_R$. Analyze these cases in the parton model. Ignore antiquarks and s, c, ... Then the proton state p_R is described by four parton distribution functions:

$$f_{uR}(x) \quad f_{uL}(x) \quad f_{dR}(x) \quad f_{dL}(x) \tag{1}$$

corresponding to quark partons with flavor u, d and with spin parallel or antiparallel to the spin of the proton.

   (a) Derive expressions, within the parton model, for the cross sections

$$\frac{d\sigma}{dxdy}(e_R^- p_R) \qquad \text{and} \qquad \frac{d\sigma}{dxdy}(e_L^- p_R) \tag{2}$$

   (b) The cross sections for e^- scattering from p_L are related to these by parity. What is the relation of the pdfs for p_L to the polarized pdfs defined in (1)? Compute the polarized deep inelastic scattering cross sections as in (2) for p_L. Check that the average over all initial state spins gives the expression for the deep inelastic scattering cross section derived in class.

   (c) When one quark has most of the momentum of the proton, we discussed in class that this is likely to be a u quark with spin parallel to the proton spin direction. (The other two quarks form an $I = 0$, $J = 0$ system with small momentum fraction.) What prediction does this model make for the limiting forms of the cross sections computed in (a) as $x \to 1$?

2. The Drell-Yan process is the creation of a $\mu^+\mu^-$ pair in high energy proton-proton collisions by the annihilation of a quark parton in one proton with an antiquark parton in the other proton. We will analyze this process in more detail in the next problem set. Here, we will use the discussion in the lecture to compute the probabilities of photon and gluon emission from the initial state q and $\overline{q}$ before they annihilate.

   (a) In the lecture, we derived eqs. (3.38) in the notes, giving the probability that an outgoing, highly relativistic, quark emits an approximately collinear photon. The same formula, it turns out, described the radiation of a photon from an initial-state quark in a hard-scattering process. The formula is a double integral, dominated by a term with two large logarithms. Find an expression for the probability that a Drell-Yan event with a muon pair of mass M also contains a radiated photon with momentum transverse to the beam direction greater than p_T. The needed limits of

integration can be obtained from the following considerations: For the $q_\perp$ integral, the radiated photon is no longer approximately collinear if $q_\perp > M/2$. For the z integral, the photon is no longer approximately collinear if the longitudinal momentum of the photon, approximately $zM/2$ in the parton-parton center of mass frame, is less than $q_\perp$. We are making an estimate, so you need keep only the leading logarithmic terms in the integral.

(b) Evaluate this expression for some typical parameters of the Drell-Yan cross section measurement at the LHC: $M = 300$ GeV, $p_T = 30$ GeV.

(c) Use eqs. (3.41) in the notes to find, in a similar way, the probability for a Drell-Yan event to contain a radiated gluon with transverse momentum greater than p_T. In this case, the final state will contain a $\mu^+\mu^-$ pair and the gluon jet.

(d) Evaluate this expression for $M = 300$ GeV, $p_T = 30$ GeV, using $\alpha_s = 0.2$. What is the probability that one of these events will contain a gluon jet?