

Physics 152/252 – Problem Set # 4

(due Thursday, April 28)

1. In the discussion in class of electron-quark scattering, we derived expressions for the needed scattering amplitudes by using results from $e^+e^- \rightarrow q\bar{q}$ plus crossing symmetry. We found

$$\begin{aligned} |\mathcal{M}(e_R^- q_R \rightarrow e_R^- q_R)|^2 &= 4Q_f^2 e^4 s^2/t^2 \\ |\mathcal{M}(e_R^- q_L \rightarrow e_R^- q_L)|^2 &= 4Q_f^2 e^4 u^2/t^2 \end{aligned} \quad (1)$$

Check these results by deriving them directly. Treat both the electron and the quark as massless fermions.

- (a) Draw the Feynman diagram for electron-quark scattering and argue that

$$\mathcal{M} = Q_f e^2 \langle e^-(p') | \bar{\psi}_e \gamma^\mu \psi_e | e^-(p) \rangle \frac{1}{t} \langle q^-(k') | \bar{\psi}_q \gamma_\mu \psi_q | q^-(k) \rangle, \quad (2)$$

where p, p' are the initial and final electron momenta and k, k' are the initial and final quark momenta.

- (b) Draw a diagram showing the kinematics of the process. Work in the center of mass frame, with the electron and the quark having initial energy E . Take the initial electron and quark directions to be along the \hat{z} axis and the final electron and quark directions to be along the vector $\hat{n} = \cos\theta\hat{z} + \sin\theta\hat{1}$. Write out the four momentum 4-vectors. Write the values of s, t , and u .
- (c) Show that the spinors with spin up and down along the direction \hat{n} are

$$\xi_+ = \begin{pmatrix} \cos\theta/2 \\ \sin\theta/2 \end{pmatrix} \quad \xi_- = \begin{pmatrix} -\sin\theta/2 \\ \cos\theta/2 \end{pmatrix}. \quad (3)$$

- (d) Construct the initial and final spinors $u(p), u(k), u(p'), u(k')$ for the electron-quark scattering reaction, both for $e_R^- q_R \rightarrow e_R^- q_R$ and for $e_R^- q_L \rightarrow e_R^- q_L$.
- (e) Compute the matrix elements

$$\langle e^-(p') | \bar{\psi}_e \gamma^\mu \psi_e | e^-(p) \rangle \quad (4)$$

for the cases of right- and left-handed electrons. There are 4 cases, but 2 are zero.

- (f) Compute the matrix elements

$$\langle q(k') | \bar{\psi}_q \gamma^\mu \psi_q | q(k) \rangle \quad (5)$$

for the two nonzero cases.

- (g) Compute the matrix elements in (1) and verify the results given.
2. The spectroscopy of mesons and baryons tells us that quarks are spin 1/2 particles, but we can also check this from the angular distribution in $e^+e^- \rightarrow$ hadrons. To analyze this, consider the alternative hypothesis that quarks are spin 0 particles. Consider their electrodynamic interactions at very high momentum where masses can be neglected.
- (a) The matrix element for the creation of a spin 0 particle of charge 1 and mass m and its antiparticle by the electromagnetic current has the form

$$\langle \phi^-(p_-)\phi^+(p_+) | e j^\mu(x) | 0 \rangle = e(p_- - p_+)^\mu e^{+ip_- \cdot x + ip_+ \cdot x} . \quad (6)$$

To justify this, note that the right-hand side of this equation must be a 4-vector built from the boson momenta p_-^μ and p_+^μ . Show, using current conservation ($\partial_\mu j_{EM}^\mu = 0$), that the structure $(p_- + p_+)^\mu$ cannot appear. Note that p_- and p_+ are on shell, i.e., $p_-^2 = p_+^2 = m^2$.

- (b) Draw the Feynman diagram for $e^+e^- \rightarrow \phi^+\phi^-$. Write the expression for the matrix element for this process the cases $e_R^- e_L^+$ and $e_L^- e_R^+$. You will need (5) and the matrix elements of the electromagnetic current between electron states, which can be found in the lecture notes, Part II, eqs. (1.31), (1.35).
- (c) Draw a diagram showing the kinematics of the process. Work in the center of mass frame, with the electron and the spin 0 boson having initial energy E . Take the initial electron and positron directions to be along the \hat{z} axis and the final boson directions to be along the vector $\hat{n} = \cos\theta\hat{z} + \sin\theta\hat{1}$. Write out the four momentum 4-vectors.
- (d) Evaluate the matrix elements from part (b), square them, and compute the differential cross section for $e^+e^- \rightarrow \phi^-\phi^+$, averaged over initial spins. Compare to the result for spin 1/2 discussed in class.
- (e) Compute the total cross section for $e^+e^- \rightarrow \phi^-\phi^+$. Show that, for $m = 0$, this is 1/4 of the corresponding result for e^+e^- annihilation to spin 1/2 particles.
3. The vector mesons ρ^0 , ω , and ϕ can decay to e^+e^- or to $\mu^+\mu^-$. The decay rates to e^+e^- are better known, since these can be measured from the inverse processes $e^+e^- \rightarrow \rho^0, \omega, \phi$.
- (a) In the Particle Data Group tables, look up the total widths of the vector mesons and their branching ratios to e^+e^- . Compute the partial decay widths of the three vector mesons to e^+e^- .
- (b) To understand the relative sizes of these widths, we will need to construct the quark model wavefunctions of the three vector mesons. Here is a mathematical warm-up exercise: For any group G , let $\{|a\rangle\}$ be basis states for a representation, and let $\{|\bar{a}\rangle\}$ be basis states for the complex conjugate representation. The

generators of the group act on the states of the representation by

$$T^i |a\rangle = (t^i)_{ab} |b\rangle \quad (7)$$

For example, $SU(2)$ acts on spinors by

$$J^i |a\rangle = \left(\frac{\sigma^i}{2}\right)_{ab} |b\rangle \quad (8)$$

Then the action on the complex conjugate representation is

$$T^i |\bar{a}\rangle = (-t^{iT})_{ab} |\bar{b}\rangle \quad (9)$$

where T denotes the matrix transpose: $(t^{iT})_{ab} = (t^i)_{ba}$. Verify this by showing that

$$T^i \left(\sum_a |a\rangle |\bar{a}\rangle \right) = \sum_a (T^i |a\rangle) |\bar{a}\rangle + |a\rangle (T^i |\bar{a}\rangle) = 0 \quad (10)$$

Then the state

$$\sum_a |a\bar{a}\rangle \quad (11)$$

is invariant under G , as it should be.

- (c) Now write the quark model flavor wavefunctions ρ^0 , ω^0 , ϕ^0 , analogous to the wavefunction

$$|\rho^+\rangle = |u\bar{d}\rangle \quad (12)$$

For ϕ , this is easy. For ρ^0 and ω^0 , you should write different linear combinations of $|u\bar{u}\rangle$ and $|d\bar{d}\rangle$. To obtain the correct combinations, you will need to use the fact that ω^0 is an isospin 0 state, while ρ^0 is part of an isospin 1 multiplet.

- (d) The matrix element for a vector meson to decay to e^+e^- is proportional to

$$\langle 0 | j_{EM}^\mu(0) | V^0 \rangle, \quad (13)$$

where $V = \rho, \omega, \phi$ and

$$j_{EM}^\mu = \sum_{f=u,d,s} Q_f \bar{\psi}_f \gamma^\mu \psi_f \quad (14)$$

Work out the relative size of the matrix elements (13) for ρ , ω , ϕ , using the approximation that the three quarks u , d , s have the same masses and strong-interaction dynamics and differ only in their electric charges. Notice that the u and d quark terms in j_{EM}^μ both contribute in the ρ and ω cases, with a different sign for the interference in the two cases.

- (e) In this same approximation, find the ratios of the decay rates. Compare to the ratios of the partial widths found in (a).