Physics 152/252 – Problem Set # 3

(due Thursday, April 21)

1. The quark model gives a theory of the magnetic moments of the proton and neutron. If a quark were an elementary Dirac fermion, its magnetic moment would be

$$\vec{\mu} = g\frac{Q_f e\hbar}{2m_f}\vec{S} \tag{1}$$

with Q_f the quark charge, m_f the quark mass, $\vec{S}$ the quark spin. The Dirac equation predicts the value of the Landé g-factor $g = 2$. In the proton and neutron, we have only the u and d quarks. By isospin symmetry, the u and d quark masses should have the same value, $m_q \approx 300$ MeV. We could then model the baryon magnetic moment as the sum of the three quark magnetic moments,

$$\vec{\mu}_B = \frac{2Q_1 e\hbar}{2m_q}\vec{S}_1 + \frac{2Q_2 e\hbar}{2m_q}\vec{S}_2 + \frac{2Q_3 e\hbar}{2m_q}\vec{S}_3 \ , \tag{2}$$

where Q_1, Q_2, Q_3 = +2/3 or -1/3 depending on whether the quark is u or d.

(a) Using the quark model wavefunction for the proton state with $S^3 = \frac{1}{2}$ written down in eq. (5.55) of the class notes, compute the magnetic moment of the proton in this approximation. This is most easily done by computing the diagonal matrix element of the $\hat{3}$ component of the operator $\vec{\mu}_B$, given by (2), in this state. Express the result by computing the proton g factor g_p given by

$$\vec{\mu}_p = g_p\frac{e\hbar}{2m_p}\vec{S}_p \tag{3}$$

and the ratio m_p/m_q.

(b) Using the same method, compute the g factor of the neutron, defined by

$$\vec{\mu}_n = g_n\frac{e\hbar}{2m_p}\vec{S}_n \tag{4}$$

(c) The g factors for the proton and neutron are very different from the value 2 predicted by the Dirac equation. The measured values are

$$g_p = +5.586 \qquad g_n = -3.826$$

Using the results from parts (a) and (b), compute these values in the quark model and compare to the measured values.

2. In class, we simplified the expression for 2-body relativistic phase space. In this problem, we will simplify the expression for 3-body phase space and make an application of that expression. In this problem, 1, 2, 3 will represent three particles with nonzero masses m_1, m_2, m_3, and $Q = p_1 + p_2 + p_3$. In the center of mass (CM) frame, $Q = (E_{CM}, 0, 0, 0)$. Let E_1, E_2, E_3 be the energies of the three particles in this frame.

   (a) Define
   $$x_1 = \frac{2Q \cdot p_1}{Q^2} \ , \quad x_2 = \frac{2Q \cdot p_2}{Q^2} \ , \quad x_3 = \frac{2Q \cdot p_3}{Q^2} \tag{5}$$
   Evaluate these quantities in the CM frame and show that
   $$x_1 + x_2 + x_3 = 2 \tag{6}$$

   (b) Write expressions for the CM energies E_i and the CM momentum values p_i in terms of the x_i, $i = 1, 2, 3$.

   (c) Show that the invariant mass of the system of particles 1 and 2 is related to x_3 by
   $$m_{12}^2 = (p_1 + p_2)^2 = (1 - x_3)Q^2 + m_3^2 \tag{7}$$
   There is a similar relation for m_{23}^2 and m_{31}^2.

   (d) Let θ_{12} be the angle between the momenta of 1 and 2 in the CM frame. Show that the formula (6) determines θ_{12} as a function of the x_i. In fact, the whole configuration of final state momenta is specified, up to an overall rotation, when the x_i are fixed.

   (e) Write out the integral over 3-body phase space in the CM frame. There are 9 integrals and 4 delta functions. Three of these delta functions can be removed by integrating out $\vec{p}_3$. Write the resulting expression as an integral over p_1, p_2 and 4 angles, constrained by 1 remaining delta function.

   (f) Because we have eliminated $\vec{p}_3$ in terms of $\vec{p}_1$ and $\vec{p}_2$, the quantity E_3 in the delta function depends $|\vec{p}_1 + \vec{p}_2|$ and therefore on $\cos\theta_{12}$. Do the integral over $\cos\theta_{12}$, eliminating the last delta function.

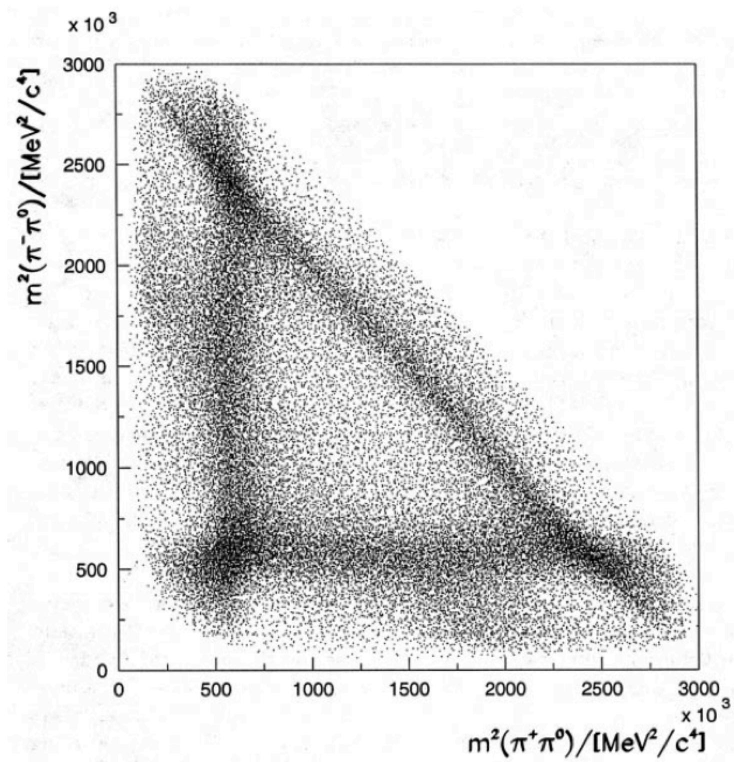
   (g) The remaining three angles simply rotate the overall configuration of momenta. Integrate over these variables.

   (h) All that remains are integrals over p_1 and p_2. Using (a), convert these to integrals over x_1 and x_2. Then, using (b), convert these to integrals over m_{23}^2 and m_{13}^2. You should find
   $$\int d\Pi_3 = \frac{Q^2}{128\pi^3} \int dx_1 dx_2 = \frac{1}{128\pi^3 Q^2} \int dm_{23}^2 dm_{13}^2 \tag{8}$$

   (i) It is amazing that the integrand has no dependence on x_1, x_2, x_3! Dalitz suggested that, for a 3-body decay $A \to 1+2+3$, we should make a scatter plot of events in the plane of m_{23}^2 vs. m_{13}^2. If the matrix element is constant, the data points will scatter evenly over this plane. Write a formula for $\Gamma(A \to 1+2+3)$ and justify this statement. If there is a resonance, that will be apparent as a clustering of points in some region. The plot of m_{23}^2 vs. m_{13}^2 is called the *Dalitz plot*.

(j) The integral in (4) should be taken over all kinematically allowed values. It takes a little work to find the boundary of the integration region. Study this first for the case in which a particle of mass M decays to three particles all of which are massless. In this case, there are allowed configurations all the way out to the boundaries $m_{13}^2 = 0$, $m_{23}^2 = 0$, $m_{12}^2 = 0$. Draw the region of integration on the (m_{13}^2, m_{23}^2) plane. For each segment of the the boundary, draw a typical momentum configuration. You should find that the boundaries of the Dalitz plot are given by configurations in which two momentum vectors are collinear and the third is directly opposite, balancing the momentum.

(k) Now consider the case of the decay of a particle of mass M to three particles with $m_1 = m_2 = 0$, $m_3 = m > 0$. Again, the boundaries of the Dalitz plot are given by configurations in which two momentum vectors are collinear and the third is directly opposite. Work out the positions of the boundaries in the (m_{13}^2, m_{23}^2) plane. The kinematic formula (2.15) in the notes will be helpful, as will the result in part (c) above.



(l) The figure above shows the Dalitz plot for a process in $p\overline{p}$ annihilation at rest,

$$p\overline{p} \to \pi^+\pi^-\pi^0 \tag{9}$$

from A. Abele, *et al.*, Phys. Lett. **B 469**, 270 (1999). Resonances are apparent. Identify the resonances as specific hadrons.

(m) The following figure shows the Dalitz plot for the decay

$$D^0 \to K^-\pi^+\pi^0 \tag{10}$$

from S. Kopp, *et al.*, Phys. Rev. **D 63**, 092001 (2001). I hope you can make out a heavy horizontal band across the lower part of the plot, a vertical band on the left, and a diagonal band on the right. These bands are obscured by the fact that interference effects cause the bands to be dark in some places but light (zero) in others. Identify these bands as specific hadrons.