Physics 152/252 – Problem Set # 2

(due Thursday, April 14)

1. This problem will give you a chance to dip into the tables of elementary particle properties produced by the Particle Data Group, available at `http://pdg.lbl.gov`, and to use this information to understand better the systematics of ψ family particle decays.

   To work this problem, you should recall that a decay rate in quantum mechanics is given by a *partial width* $\Gamma(A \to f)$, with units of energy. A partial width gives the rate of a basic quantum mechanical process. The *total width* of a resonance is

   $$\Gamma_A = \sum_f \Gamma(A \to f) \tag{1}$$

   That is, it is the sum of the rates for all possible decay processes. The lifetime of the resonance is $\tau = \hbar/\Gamma_A$. The *branching ratio* to the decay channel f, the probability that a particular decay of A gives the final states f, is

   $$BR(A \to f) = \Gamma(A \to f)/\Gamma_A \ . \tag{2}$$

   Usually, it is easiest to meaure branching ratios, but the real physics is in the actual rates. To get these, we must extract the partial widths from the information that we are given.

   (a) The J/ψ can decay in four different ways. (1) decay by $c\bar{c}$ annihilation directly to hadrons, (2) decay by $c\bar{c}$ annihilation to a virtual photon (a short-lived state of electromagnetic fields), which then materializes into an e^+e^- or $\mu^+\mu^-$ pair. The J/ψ is produced in e^+e^- annihilation by e^+e^- annihilation into a virtual photon which then materializes as a J/ψ. This decay is the reverse of that process, (3) decay by $c\bar{c}$ annihilation to a virtual photon, which then materializes into hadrons, (4) decay to 1 photon plus hadrons. There is also a decay to 3 photons with a very small branching ratio (about 10^{-5}).

   Look up the listing for the J/ψ at the Particle Data Group web site. The heading "pdgLive" gives the most recently updated information. Look under $c\bar{c}$ to find the information for the J/ψ. The entry $J/\psi \to ggg$ gives the branching ratio for direct decays to hadrons, mode (1) above. Similarly, the entry $J/\psi \to \gamma gg$ gives the branching ratio for mode (4) above.

   Write the branching ratio for each of the decay modes (1)–(4). (These should add up to 100%, within the measurement errors.) Using the tabulated total width, find the partial width for each channel.

(b) The $\psi(2S)$ can decay by the 4 modes above and also by 3 additional modes: (5) decay to the heavy lepton $\tau^+\tau^-$, (6) decay to J/ψ plus hadrons ($\pi\pi$, π^0, or η), (7) radiative decay to the 1P states χ_c.

Using the information in the entry for the $\psi(2S)$, write the branching ratio for each of the decay modes (1)–(7). (Again, these should add up to 100%, within the measurement errors.) Using the tabulated total width, find the partial width for each channel.

(c) Compute the ratios of the partial widths between the J/ψ and the $\psi(2S)$ for each of the processes (1)–(4). How do these ratios compare? Why would this result be expected?

2. Consider the reaction of pion-nucleon scattering at energies of a few hundred MeV. Two prominent resonances are seen as the center of mass energy is varied. These are the Δ resonances at 1232 MeV and the N^* ("Roper") resonance at 1440 MeV. The Δ has $I = \frac{3}{2}$, $S = \frac{3}{2}$, as was explained in class. The Roper has $I = \frac{1}{2}$, $S = \frac{1}{2}$ and can be thought of as a radial excitation of the nucleon. The absolute rates of the reactions that form these resonances need to be computed from a dynamical strong interaction theory. However, the relative rates of different reactions producing the same resonances can be computed using isospin symmetry and Clebsch-Gordan coefficients.

The initial states in the reaction are the π mesons, an $I = 1$ multiplet $\pi^-, \pi^0, \pi^+)$, and the nucleons, an $I = \frac{1}{2}$ multiplet $N = (p, n)$. The quantum mechanical amplitude to produce a resonance of isospin I from initial states with isospins (I_1, I_1^3) and (I_2, I_2^3) is proportional to the Clebsch-Gordan coefficient

$$\langle I_1 I_2 I_1^3 I_2^3 \,|\, I_1 I_2 I I^3 \rangle \tag{3}$$

with $I^3 = I_1^3 + I_2^3$. The amplitude for the decay of a resonance to two particles of definite isospin is similarly proportional to the relevant Clebsch-Gordan coefficient. You can find a very readable table of Clebsch-Gordan coefficients for $SU(2)$ at the Particle Data Group web site, under "Mathematical Tools".

(a) There are 4 Δ states: $(\Delta^{++}, \Delta^+, \Delta^0, \Delta^-)$. These decay exclusively to 2-particle states πN. Using isospin Clebsch-Gordan coefficients, compute the branching ratios for each state to the 6 possible channels

$$(\pi^+\pi^0, \pi^-) \times (p, n)$$

(b) A crude description of the N^* decays is that 60% of the decays go to πN and 40% go to $\pi\Delta$, Using these values and the Clebsch-Gordan coefficients, compute the branching ratios of the N^* states (N^{*+}, N^{*0}) to the 6 πN states in (a).

(c) The decay of the N^* to $\pi\Delta$ followed by the decay of the Δ leads to the final state $\pi\pi N$. It is easy to compute the branching ratios to the various $\pi\pi N$ states if we assume that there is no quantum mechanical interference between two decay processes. (This will be correct if two pions emitted have significantly different energies, which is actually not so true in this case.) Using this approximation, compute the branching ratios of N^* to the various possible $\pi\pi N$ states.