Physics 152 / 252 - Final Exam

June 6, 2016

The exam has 100 points in all. Partial credit will be given in each question.

1. **(20 points)** The highest energy experiments at the LEP e^+e^- collider observed the process
$$e^+e^- \to W^+W^- \quad \text{and} \quad e^+e^- \to ZZ \ . \tag{1}$$
In this problem, you will discuss how these processes were recognized and discriminated.

   (a) Three distinct classes of events contribute to this signal for $e^+e^- \to W^+W^-$, since (i) both W's can decay to hadrons, (ii) both W's can decay to leptons, or (iii) one W can decay to hadrons, while the other decays to leptons. Sketch how an event of each type will appear in a collider detector. What detector elements are needed to observe each type of event?

   (b) The Z has decays to e^+e^- and $\mu^+\mu^-$, with two tracks whose momenta add to the mass of the Z. Events with two lepton tracks of this type obviously contain a Z in the final state. Find two other decay modes of the Z that are unique to this particle, in the sense that they are observably different from possible W decays. One of these modes should involve a Z decay to hadrons only.

2. **(50 points)** A possible decay of the π^+ is β decay: $\pi^+ \to \pi^0 e^+ \nu$. In this problem, you will work out the rate of this decay. For the purposes of this exam, you may regard the π^+ as infinitely heavy with respect to the leptons (with $m(\pi^+) - m(\pi^0) = \Delta m =$ 4.6 MeV). That is, assume that the final π^0 absorbs the momentum of the $e^+\nu$ but remains at rest for the purpose of computing matrix elements. You may also set the masses of the positron and the neutrino equal to zero.

   (a) Write the matrix element for the decay using V – A theory.

   (b) To evaluate the hadronic part of the matrix element, a useful reference point is
$$\left\langle \pi^+(p) \right| j^{\mu 3}(x) \left| \pi^+(p) \right\rangle = 2p^\mu \tag{2}$$
Explain the features of this equation: Why does p^μ appear? Why is the coefficient in front exactly 2? [Hint: j^{30} is the charge density of isospin I^3.]

   (c) Evaluate the hadronic part of the matrix element in (a) using (1). You might find it useful to recall that, for a spin 1 multiplet,
$$\langle m | J^3 | m \rangle = m \ , \qquad \langle m = 0 | J^- | m = 1 \rangle = \sqrt{2} \ . \tag{3}$$
This gives the contribution from $j^{\mu a}$. Is there a contribution from $j^{\mu 5a}$?

   (d) Evalulate the leptonic part of the matrix element in (a) using explicit 2-component spinors.

(e) Combine the pieces and write an explicit formula for the decay rate in terms of 3-body phase space.

(f) Integrate out the momentum of the π^0, and evaluate the rest of the phase space integrals. Find an explicit expression for the decay partial width.

(g) Find the value of this partial width, in MeV.

(h) The lifetime of the π^+ is 2.6×10^{-8} sec. What is the branching ratio of the π^+ to $\pi^0 e^+ \nu$?

3. **(30 points)** In large momentum transfer reactions, for example, in hard scattering processes at the LHC, the wavefunction of the proton contains pairs of charm quarks generated from gluons. In this problem, you will work out how many are expected.

(a) A simple form for the pdf of the gluon is

$$f_g(x) = \frac{3}{2} \frac{(1-x)^2}{x} \ . \tag{4}$$

With this expression, what fraction of the total momentum of the proton is carried by gluons? In the rest of this problem assume that the gluon pdf is independent of Q with this value and that α_s is constant with the value $\alpha_s = 0.118$. (Real gluon distributions fall off faster as $x \to 1$, as $(1-x)^a/x$, where $a \geq 5$.)

(b) Draw a Feynman diagram for the process $pp \to gc\bar{c}$ in which a gluon from one proton splits to an almost collider $c\bar{c}$ pair, and then a gluon from the other proton scatters with the c with large momentum transfer Q.

(c) Write an expression for the total number of c quarks generated by the almost-collinear splitting that are then available for scattering from the second gluon. Use the appropriate Altarelli-Parisi splitting function given below. Use the expression for $f_g(x)$ from (a). Your expression for the total number of c quarks should contain 3 integrals, one over the $p_\perp$ of the splitting, one over the z of the splitting, and one over x of the initial gluon.

(d) Integrate over z.

(e) By guessing appropriate upper and lower limits, estimate the value of the integral $\int dp_\perp / p_\perp$ in the expression in (c), in terms of Q and the mass m_c of the c quark.

(f) By guessing appropriate upper and lower limits and then evaluating the integral, estimate the value of the integral over x.

(g) Assemble the pieces. For an LHC collision energy at center of mass energy E_{CM} and given Q, write a formula for the expected number of c quarks are available in each proton for hard scattering from a gluon.

(h) Evaluate this formula numerically for $E_{CM} = 13$ TeV and $Q = 200$ GeV.

Useful Information

Pauli sigma matrices:

$$\sigma^1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \qquad \sigma^2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \qquad \sigma^3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

$$\begin{aligned} c &= 2.998 \times 10^{10} \text{ cm/sec} \\ \hbar &= 6.582 \times 10^{-22} \text{ MeV-sec} \\ (\hbar c)^2 &= 0.389 \text{ GeV}^2\text{mbarn} \quad (1 \text{ mbarn} = 10^{-27} \text{ cm}^2) \\ 1 \text{ yr} &= 3.156 \times 10^7 \text{ sec} \end{aligned} \tag{5}$$

Fermi constant: $G_F = 1.166 \times 10^{-5}$ GeV^{-2}

Masses of particles:

e	0.51 MeV	π^0	135.0 MeV
μ	106 MeV	π^+	139.6 MeV
τ	1.777 GeV	K^+	494 MeV
W	80.4 GeV	K^0	498 MeV
Z	91.2 GeV	J/ψ	3097 MeV
t	173 GeV	ψ'	3686 MeV

Masses of quarks: (at $Q = 4$ GeV)

u	2 MeV	s	85 MeV
d	4 MeV	c	1050 MeV

Coupling constants at $Q = m_Z$:

$$\alpha = 1/128 \qquad \alpha_w = 1/29.6 \qquad \alpha' = 1/98 \qquad \alpha_s = 1/8.5 \tag{6}$$

Spinors for massless fermions: for $\vec{p} \parallel (\cos\theta\hat{3} + \sin\theta\hat{1})$,

$$u_L(p) = v_L(p) = \sqrt{2p}\begin{pmatrix} -\sin\theta/2 \\ \cos\theta/2 \end{pmatrix} \tag{7}$$

where these are spinors for a left-handed fermion and a right-handed antifermion.

Altarelli-Parisi splitting functions:

$$\begin{aligned} P_{g\leftarrow q}(z) &= \frac{4}{3}(1 + (1-z)^2)/z \\ P_{q\leftarrow q}(z) &= \frac{4}{3}(1 + z^2)/(1-z) + A\delta(z-1) \\ P_{q\leftarrow g}(z) &= \frac{1}{2}(z^2 + (1-z)^2) \\ P_{g\leftarrow g}(z) &= 3(\frac{1 + z^4 + (1-z)^4}{z(1-z)}) + B\delta(z-1) \end{aligned} \tag{8}$$