Physics 152 / 252 - Final Exam

June 8, 2015

The exam has 100 points in all. Partial credit will be given in each question.

1. **(20 points)** The LHC experiments have measured the lifetime of the Λ_b baryon—the lowest energy *bud* state–to be:

$$\tau = 1.45 \times 10^{-12} \text{ sec} \tag{1}$$

The measurements use the specific Λ_b decay

$$\Lambda_b \to J/\psi + \Lambda \ , \quad J/\psi \to \mu^+\mu^- \ , \quad \Lambda \to p\pi^- \tag{2}$$

   (a) Draw a quark-level Feynman diagram showing how the $\Lambda_b \to J/\psi + \Lambda$ decay takes place. What elementary weak interaction vertices are involved?

   (b) What elements of a collider detector are used to measure the momenta or energies of the various final-state particles?

   (c) What is $c\tau$, the estimated (proper) distance that a Λ_b travels before decaying?

   (d) How is this distance measured?

   (e) The Λ decays 64% of the time to $p\pi^-$ and another 36% of the time to $n\pi^0$. Why is the $n\pi^0$ mode not suitable to be used for this lifetime measurement?

2. **(50 points)** A cosmic ray neutrino $\overline{\nu}_e$ of sufficiently high energy can annihilate with an atomic electron to produce a W^- boson.

   (a) What energy neutrino is required? Consider the electron as a massive particle at rest, and ignore the width of the W.

   (b) What is the polarization state of the incoming neutrino (assuming that it was generated in the weak-interaction decay of a hadron)?

   (c) Compute the matrix elements for the process $\overline{\nu}_e e^- \to W^-$. The matrix elements for the two electron spin states $S^3 = \pm\frac{1}{2}$, where $\hat{3}$ is the collision axis, are different. Show that the matrix element for the $S^3 = -\frac{1}{2}$ case is negligible compared to that for $S^3 = +\frac{1}{2}$.

   (d) Compute the cross section for $\overline{\nu}_e e^- \to W^-$. The result should be of the form

$$\sigma = A \ \delta(s - m_W^2)$$

where the delta function results from 1-body phase space,

$$\int d\Pi_! = \int \frac{d^3p}{(2\pi)^3 2E_p}(2\pi)^4\delta^{(4)}(P-p) = (2\pi)\delta(P^2 - M^2) \ ,$$

where P is the initial-state momentum $P = p_A + p_B$.

(e) The constant A in (d) is a dimensionless number. Evaluate it numerically.

(f) The cosmic ray neutrino flux at very high energy is (optimistically) estimated to be

$$\frac{d\Phi}{dE_\nu} \approx \frac{10^{-7}}{[E_\nu \text{ (GeV)}]^2} \text{ GeV}^{-1}\text{cm}^{-2}\text{sec}^{-1}$$

where $[E_\nu$ (GeV)] denotes the numerical value of the neutrino energy in GeV. The overall units of this quantity are GeV^{-1}·(flux units). Of this neutrino flux, about $\frac{1}{3}$ are antineutrinos and, of those, about $\frac{1}{3}$ are electron-type antineutrinos. Compute the rate for W^- production (events/sec) for each target electron.

(g) A large detector is needed. Probably, for such a detector, the only easily visible modes of W decay would be $W^- \to \mu^-\overline{\nu}$ and $W^- \to \tau^-\overline{\nu}$ For example, the IceCube detector instruments large volumes of ice at the South Pole. The very energetic leptons give characteristic signals in Cherenkov light. Estimate the volume of detector needed to produce 5 visible events per year.

(h) What is the energy distribution of the muons, in the lab frame? Write your answer as a distribution $d\sigma/dx$, where $x = E_\mu/E_W$. Don't worry about the normalization, just give the shape of this function. One way to work this out is to first write the angular distribution of the muons in the W rest frame and then boost to the lab frame. (Yes, the polarizations of the initial and final particles are relevant to this answer.)

3. **(30 points)** One of the goals of the LHC experiments is to discover the production of very weakly interacting particles that might make up the dark matter of the universe. Obviously, if these partiucles are very weakly interacting, they will not make signals in the LHC detectors.

(a) Assume that pairs of dark matter particles χ of mass 200 GeV are produced in pp collisions in a reaction

$$q\overline{q} \to \chi\overline{\chi}$$

Using Altarelli-Parisi evolution, estimate the fraction of $\chi\overline{\chi}$ events that also produce a gluon of $p_T > 50$ GeV. It is sufficient to write down and estimate the dominant logarithmic terms. Igmore the running of α_s and simply take $\alpha_s = 1/8.4$.

(b) Eastimate the fraction of $\chi\overline{\chi}$ events that also produce a photon with $p_T > 50$ GeV. As in (a), it is sufficient to write down and estimate the dominant logarithmic terms. Assume for simplicity that all of the reactions involve valence quarks annihilating sea antiquarks.

(c) There is a Standard Model reaction that also produces a high-p_T gluon or photon together with invisible particles. What is it? Draw the relevant Feynman diagrams.

Useful Information

Pauli sigma matrices:

$$\sigma^1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \qquad \sigma^2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \qquad \sigma^3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

$$\begin{aligned} c &= 2.998 \times 10^{10} \text{ cm/sec} \\ \hbar &= 6.582 \times 10^{-22} \text{ MeV-sec} \\ (\hbar c)^2 &= 0.389 \text{ GeV}^2\text{mbarn} \quad (1 \text{ mbarn} = 10^{-27} \text{ cm}^2) \\ 1 \text{ au} &= 1.661 \times 10^{-24} \text{ g} = (\text{mass of } {}^{12}\text{C})/12 \\ 1 \text{ yr} &= 3.156 \times 10^7 \text{ sec} \end{aligned} \tag{3}$$

Masses of particles:

e	0.51 MeV	π^0	135 MeV
μ	106 MeV	π^+	140 MeV
τ	1.777 GeV	K^+	494 MeV
W	80.4 GeV	K^0	498 MeV
Z	91.2 GeV	J/ψ	3097 MeV
t	173 GeV	ψ'	3686 MeV

Coupling constatnts at $Q = m_Z$:

$$\alpha = 1/128 \qquad \alpha_w = 1/30 \qquad \alpha' = 1/98 \qquad \alpha_s = 1/8.4 \tag{4}$$

Spinors for massless fermions: $(\vec{p} \parallel \cos\theta \hat{3} + \sin\theta \hat{1})$

$$u_L(p) = v_L(p) = \sqrt{2p} \begin{pmatrix} -\sin\theta/2 \\ \cos\theta/2 \end{pmatrix} \tag{5}$$

where these are spinors for a left-handed fermion and a right-handed antifermion.

Altarelli-Parisi splitting functions:

$$\begin{aligned} P_{g\leftarrow q}(z) &= \frac{4}{3}(1 + (1-z)^2)/z \\ P_{q\leftarrow q}(z) &= \frac{4}{3}(1 + z^2)/(1-z) + A\delta(z-1) \\ P_{q\leftarrow g}(z) &= \frac{1}{2}(z^2 + (1-z)^2) \\ P_{g\leftarrow g}(z) &= 3(\frac{1 + z^4 + (1-z)^4}{z(1-z)}) + B\delta(z-1) \end{aligned} \tag{6}$$