Physics 152 / 252 - Final Exam

June 10, 2014

1. (20 points) In future experiments, we expect to observe the reaction

$$e^+e^- \to t\bar{t} \tag{1}$$

with

$$t \to W^+b \qquad \bar{t} \to W^-\bar{b} \tag{2}$$

For definiteness, consider such events at a center of mass energy of 1000 GeV.

   (a) Show that events of this type can end up with either 2, 4, or 6 quarks and antiquarks in the final state. Estimate the probabilities of each outcome. Ignore QCD corrections.

   (b) For an event with 4 quarks or antiquarks and a μ^+ in the final state, draw a typical event display in a collider detector. Indicate what particles are observed, and by which detector elements.

2. (30 points) The ψ' can decay to the J/ψ via the process

$$\psi' \to J/\psi + 2\pi \tag{3}$$

If the two pions are in the state $\left|\pi^a(p_1)\pi^b(p_2)\right\rangle$, where a, b are isospin indices $1, 2, 3$, the matrix element for this process has the form

$$\mathcal{M} = A\, p_{1\mu}p_2^\mu\, \delta^{ab} \tag{4}$$

The branching ratio to this channel is 51%.

   (a) Justify the isospin structure of $\mathcal{M}$. What does this predict for the ratio $BR(\psi' \to \psi\pi^+\pi^-)/BR(\psi' \to \psi\pi^0\pi^0)$?

   (b) The form for the matrix element follows from the assumption that the matrix element vanishes if $p_1 = 0$ or $p_2 = 0$. This is expected in the approximation that the pion mass is zero. Why?

   (c) The decay $\psi' \to J/\psi\pi^0$ is seen, but at a suppressed rate. Why is it suppressed? Estimate the branching ratio.

   (d) Write a formula for the decay rate for $\psi' \to J/\psi + 2\pi$ in terms of the parameter A. Use the simplifying assumptions that the pions have zero mass and that the ψ particles are very heavy: $m(\psi') \gg \Delta m = [m(\psi') - m(J/\psi)]$. How does the decay rate depend on the mass difference Δm?

   (e) Evaluate the integrals in (d) and write a compact formula for the decay rate $\Gamma(\psi' \to J/\psi + 2\pi)$.

3. (50 points) The dominant decay of the π^0 is $\pi^0 \to \gamma\gamma$. This decay has a matrix element of the form

$$\mathcal{M}(\pi^0 \to \gamma(p_1 \epsilon_1)\gamma(p_2, \epsilon_2) = A\epsilon^{\mu\nu\lambda\sigma} p_{1\mu} p_{2\nu} \epsilon^*_{1\sigma} \epsilon^*_{2\lambda} \qquad (5)$$

where $\epsilon^{\mu\nu\lambda\sigma}$ is the totally antisymmetric tensor with $\epsilon^{0123} = 1$.

(a) What is the relation between the polarization vectors of the two final photons implied by (5)? It is easiest to discuss this in terms of linear polarization.

(b) Why is the simpler structure $\epsilon^*_{1\mu}\epsilon_2^{*\mu}$ not allowed?

(c) The decay $\pi^0 \to e^+e^-$ is allowed but is very rare, with a branching ratio of 10^{-8}. Why is this process suppressed?

(d) The process $\pi^0 \to \gamma e^+e^-$ is called the *Dalitz decay* of the π^0. Draw a Feynman diagram contributing to this process.

(e) Compute the matrix element

$$\mathcal{M}(\gamma_L(p) \to e^-_L(q)e^+_R(k)) \qquad (6)$$

needed for this process. Take $\vec{p} \parallel \hat{3}$. Use the approximation that the γ splits to a nearly collinear e^+e^- pair. You may set the mass of the electron to zero in this calculation.

(f) Write the other 3 nonzero matrix elements of this type, with different polarizations.

(g) Write a formula for the partial width of the Dalitz decay of the π^0, in terms of the partial width $\Gamma(\pi^0 \to \gamma\gamma)$ and integrals over the final e^+e^- coordinates. You can use the result of parts (d), (e). One of the integrals will be infinite in the limit of zero electron mass.

(h) Approximate the integral appropriately to the case in which the electron mass is small but not exactly zero. How does the partial width depend on the electron mass?

(i) Give a numerical estimate for the branching ratio for the Dalitz decay of the π^0.

Useful Information

Pauli sigma matrices:

$$\sigma^x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \qquad \sigma^y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \qquad \sigma^z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

Masses of particles:

e	0.51 MeV	π^0	135 MeV
μ	106 MeV	π^+	140 MeV
τ	1.777 GeV	K^+	494 MeV
W	80.4 GeV	K^0	498 MeV
Z	91.2 GeV	J/ψ	3097 MeV
t	173 GeV	ψ'	3686 MeV

Spinors for massless fermions: $(\vec{p} \parallel \cos\theta\hat{3} + \sin\theta\hat{1})$

$$u_L(p) = v_L(p) = \sqrt{2p}\begin{pmatrix} -\sin\theta/2 \\ \cos\theta/2 \end{pmatrix} \tag{7}$$

where these are spinors for a left-handed fermion and a right-handed antifermion.