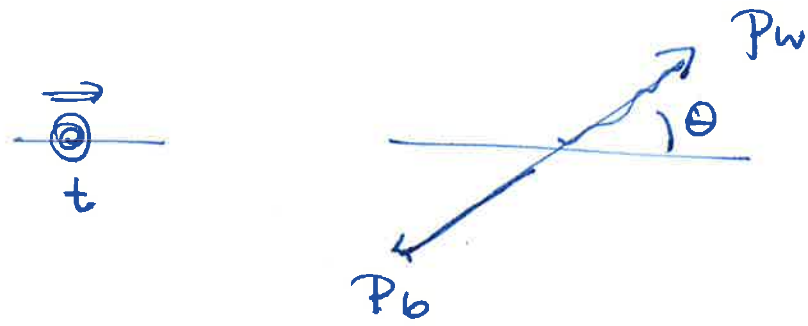
Physics 152/252

Problem Set #9 – Solutions

1) a.



$$P_W = (E_W,\ P_W \sin\theta,\ 0\ \ P_W\cos\theta)$$

$$P_b = (P_W,\ -P_W\sin\theta,\ 0,\ -P_W\cos\theta)$$

$$u_t = \sqrt{m_t}\begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} \qquad u_b = \sqrt{2P}\begin{pmatrix} \eta \\ 0 \\ 0 \end{pmatrix}$$

where η is the left handed spinor in the b direction

$$\eta = \begin{pmatrix} -\cos\theta/2 \\ -\sin\theta/2 \end{pmatrix}$$

$$E_W = \frac{m_t^2 + m_W^2}{2m_t} \qquad P_W = \frac{m_t^2 - m_W^2}{2m_t}$$

b.) $$\mathcal{M} = \frac{g}{\sqrt{2}}\sqrt{2p}\,(-\cos\theta/_2, -\sin\theta/_2)\,(1,-\sigma^1,-\sigma^2,-\sigma^3)^\mu \sqrt{m_t}\begin{pmatrix}1\\0\end{pmatrix}$$

$$\cdot\ \varepsilon^*_{W\mu}$$

$$= \frac{g}{\sqrt{2}}\sqrt{2m_t p}\,(-\cos\theta/_2, \sin\theta/_2, i\sin\theta/_2, \cos\theta/_2)^\mu$$

$$\cdot \frac{1}{\sqrt{2}}(0, \cos\theta, +i, -\sin\theta)_\mu$$

$$= \frac{g}{2}\sqrt{2m_t p}\,(-1)\left[0 + \sin\theta/_2\cos\theta - \sin\theta/_2 - \sin\theta\cos\theta/_2\right]$$

$$= \frac{g}{2}\sqrt{2m_t p}\,\left[\sin\theta/_2 + (\sin\theta\cos\theta/_2 - \sin\theta/_2\cos\theta)\right]$$

$$= g\sqrt{2m_t p}\,\sin\theta/_2$$

$$= g\,(m_t^2 - m_W^2)^{1/2}\sin\theta/_2$$

$$\Gamma(t \to bW^+_-) = \frac{1}{2m_t}\frac{1}{8\pi}\underbrace{\left(1-\frac{m_W^2}{m_t^2}\right)}_{\frac{2p}{m_t}}\underbrace{\int_{-1}^{1}\frac{d\cos\theta}{2}\sin^2\theta/_2}_{\frac{1}{2}}\cdot g^2(m_t^2-m_W^2)$$

$$= \frac{g^2}{32\pi}\,m_t\left(1-\frac{m_W^2}{m_t^2}\right)^2$$

$$= \frac{\alpha_W}{8}\,m_t\left(1-\frac{m_W^2}{m_b^2}\right)^2.$$

c.) $$\mathcal{M} = \frac{g}{\sqrt{2}}\sqrt{2pm_t}\,\left(-\cos\theta/2,\ \sin\theta/2,\ i\sin\theta/2,\ \cos\theta/2\right)^\mu \cdot \frac{1}{\sqrt{2}}\left(0,\ \cos\theta,\ -i,\ -\sin\theta\right)_\mu$$

$$= \frac{g}{2}\sqrt{2m_t p}\left[0 + \sin\theta/2\cos\theta + \sin\theta/2 - \sin\theta\cos\theta/2\right]$$

$$= 0$$

For a positive helicity W^+, the final state is

which has $J^3 = +3/2$, therefore $J \geq 3/2$. So a spin ½ t quark cannot decay to this state.

d.) In the top quark rest frame

$$\varepsilon_0^\mu = (0\ 0\ 0\ 1)$$

The boost of this vector is

$$\varepsilon_0^\mu = \gamma(\beta, 0, 0, 1)$$

$$\gamma = \frac{E_W}{m_W} \qquad \beta\gamma = \frac{p_W}{m_W}$$

e.) $$\varepsilon_0 = \left(\frac{P}{m_W}, 0\ 0\ \frac{E}{m_W} \right)$$

$$\xrightarrow{\text{rotate}} \left(\frac{P}{m_W}, \frac{E}{m_W} \sin\theta, 0, \frac{E}{m_W} \cos\theta \right)$$

then

$$\mathcal{M} = \frac{g}{\sqrt{2}} \sqrt{2 p m_t} \left(-\cos\theta/2, \sin\theta/2, i \sin\theta/2, \cos\theta/2 \right)^\mu$$

$$\cdot \left(\frac{P}{m_W}, \frac{E}{m_W} \sin\theta, 0, \frac{E}{m_W} \cos\theta \right)_\mu$$

$$= \frac{g}{\sqrt{2}} \sqrt{2 p m_t} \left[-\frac{P}{m_W} \cos\theta/2 - \frac{E}{m_W} \left(\sin\theta \sin\theta/2 + \cos\theta \cos\theta/2 \right) \right]$$

$$= -\frac{g}{\sqrt{2}} \sqrt{2 p m_t} \left(\frac{P+E}{m_W} \right) \cos\theta/2$$

$$= -\frac{g}{\sqrt{2}} (2 p m_t)^{1/2} \left(\frac{m_t}{m_W} \right) \cos\theta/2$$

then

$$\Gamma(t \to b W_0^+) = \frac{1}{2 m_t} \frac{1}{8\pi} \left(1 - \frac{m_W^2}{m_t^2} \right) \left(\underbrace{\int \frac{d\cos\theta}{2} \cos^2\theta/2}_{1/2} \right) \frac{g^2}{2} (m_t^2 - m_W^2) \frac{m_t^2}{m_W^2}$$

$$= \frac{\alpha_W}{16} m_t \left(\frac{m_t}{m_W} \right)^2 \left(1 - \frac{m_W^2}{m_t^2} \right)^2$$

f.) $\Gamma(t) = \frac{\alpha_W}{16} m_t \left(\frac{m_t^2}{m_W^2}\right)\left(1 + 2\frac{m_W^2}{m_t^2}\right)\left(1 - \frac{m_W^2}{m_b^2}\right)^2$

g) $\frac{\Gamma(t\to bW_0^+)}{\Gamma(t\to bW_-^+)} = \frac{\left(\frac{1}{2}m_t^2/m_W^2\right)}{1} = \frac{m_t^2}{2m_W^2} = 2.3$

h.) $\frac{y_t^2}{g^2} = \left(\frac{\sqrt{2}m_t/v}{2m_W/v}\right)^2$ $\quad m_t = \frac{y_t v}{\sqrt{2}}$

$m_W = \frac{gv}{2}$

$= \frac{m_t^2}{2m_W^2}$

$=$ result of part (g)

so in $t \to W_0^+$, the W^+ couples like a Higgs boson, not like a gauge boson.