Physics 152/252

Problem Set #5 - Solutions

1.) For a p_R, ignoring antiquarks, s, c etc., there are four relevant parton distribution functions.

$$f_{uR}(x) \qquad f_{uL}(x) \qquad f_{dR}(x) \qquad f_{dL}(x)$$

giving the probabilities for finding a u_R, u_L d_R d_L at a given value of x. The deep inelastic scattering cross section is given by

$$\sigma(e^-_{L,R}\, p_R \to e^-_{L,R} + X)$$

$$= \int_0^1 dx \sum_{f,\, s=L,R} f_{fs}(x)\, \sigma(e^-_{L,R}\, q_{fs}(xP) \to e^-_{L,R}\, q_{fs})$$

We can analyze this formula as discussed in class for the unpolarized distributions.

To derive the cross section, we used

$$\mathcal{M}(e^-_R q_R \to e^-_R q_R) = 4Q_f^2 e^4 \frac{s^2}{t^2} \qquad \text{same for } e^-_L q_L$$

$$\mathcal{M}(e^-_R q_L \to e^-_R q_L) = 4Q_f^2 e^4 \frac{u^2}{t^2} \qquad \text{same for } e^-_L q_R$$

From these expressions we find

$$\left.\frac{d\sigma}{dt}(eq \to eq)\right|_{\text{unpolarized}} = \frac{2\pi Q_f^2 \alpha^2}{s^2}\left(\frac{s^2+u^2}{t^2}\right)$$

[class notes, eq. (2.51)]

This was made up from

$$\left.\frac{d\sigma}{dt}\right|_{\text{unpol.}} = \frac{1}{4}\left[\frac{d\sigma}{dt}(\bar{e}_R q_R \to \bar{e}_R q_R) + \frac{d\sigma}{dt}(\bar{e}_R q_L \to \bar{e}_R q_L) + \frac{d\sigma}{dt}(\bar{e}_L q_R \to \bar{e}_L q_R) + \frac{d\sigma}{dt}(\bar{e}_L q_L \to \bar{e}_L q_L)\right]$$

so

$$\frac{d\sigma}{dt}(\bar{e}_R q_R \to \bar{e}_R q_R) = \frac{d\sigma}{dt}(\bar{e}_L q_L \to \bar{e}_L q_L) = \frac{4\pi Q_f^2 \alpha^2}{s^2}\,\frac{s^2}{t^2}$$

$$\frac{d\sigma}{dt}(\bar{e}_R q_L \to \bar{e}_R q_L) = \frac{d\sigma}{dt}(\bar{e}_L q_R \to \bar{e}_L q_R) = \frac{4\pi Q_f^2 \alpha^2}{s^2}\,\frac{u^2}{t^2}$$

From here, changing what needs to be changed in the notes, we find

$$\sigma(\bar{e}_R P_R) = \int dx\, dQ^2 \sum_f Q_f^2 \left\{ f_{fR}(x)\cdot\left[\frac{4\pi Q_f^2\alpha^2}{Q^4}\cdot 1\right] + f_{fL}(x)\cdot\left[\frac{4\pi Q_f^2\alpha^2}{Q^4}(1-y)^2\right]\right\}$$

or

$$\frac{d\sigma(\bar{e}_R P_R)}{dx\,dy} = \sum_f x\, Q_f^2\left[f_{fR}(x) + f_{fL}(x)(1-y)^2\right]\frac{4\pi\alpha^2 s}{Q^4}$$

and, similarly for e_L^-

$$\frac{d\sigma}{dx\,dy}(e_L^- P_R) = \sum_f x Q_f^2 \left[f_{fR}(x)\,(1-y)^2 + f_{fL}(x) \right] \frac{4\pi\alpha^2 s}{Q^4}$$

b.) Parity sends $f_L \leftrightarrow f_R$. So the pdf's of P_L are:

$u_R : \; f_{uL}(x) \qquad d_R : \; f_{dL}(x) \qquad$ where $f_{uL}(x)$ are the

$u_L : \; f_{uR}(x) \qquad d_L : \; f_{dR}(x) \qquad$ pdf's of P_R

so

$$\frac{d\sigma}{dx\,dy}(e_R^- P_L) = \sum_f x Q_f^2 \left[f_{fL}(x) + f_{fR}(x)\,(1-y)^2 \right] \frac{4\pi\alpha^2 s}{Q^4}$$

$$\frac{d\sigma}{dx\,dy}(e_L^- P_L) = \sum_f x Q_f^2 \left[f_{fL}(x)\,(1-y)^2 + f_{fR}(x) \right] \frac{4\pi\alpha^2 s}{Q^4}$$

Then the unpolarized cross section is

$$\frac{d\sigma}{dx\,dy}(ep) = \frac{1}{4}\,(\text{sum of 4 eqs above})$$

$$= \sum_f x Q_f^2 \left[f_{fR}(x) + f_{fL}(x) \right] \cdot 2\,(1 + (1-y)^2) \cdot \frac{1}{4}\, \frac{4\pi\alpha^2 s}{Q^4}$$

$$= \sum_f x Q_f^2 \left[f_{fR}(x) + f_{fL}(x) \right] \frac{2\pi\alpha^2 s}{Q^4} (1 + (1-y)^2)$$

and the pdf of a quark of flavor f in the proton is

$$f_f(x) = f_{fR}(x) + f_{fL}(x).$$

c.) As $x \to 1$, the dominant pdf is $f_{uR}(x)$

then

$$\frac{d\sigma}{dx\,dy} \underset{x\to 1}{\cong} \frac{4}{9} \times f_{uR}(x) \frac{4\pi\alpha^2 s}{Q^4}$$

$$\frac{d\sigma}{dx\,dy} \underset{x\to 1}{\cong} \frac{4}{9} \times f_{uR}(x) \frac{4\pi\alpha^2 s}{Q^4} (1-y)^2$$

2.) a) $\text{Prob}(\to \gamma) \cong \frac{\alpha Q_f^2}{\pi} \int \frac{dq_\perp}{q_\perp} \int \frac{dz}{z} [1 + (1-z)^2]$

The limits of ~~integration~~ are approximately

$$\int_{p_T}^{M/2} \frac{dq_\perp}{q_\perp} \int_{2q_\perp/M}^{1} \frac{dz}{z}$$

$$\uparrow$$

$$\frac{zM}{2} = q_\perp$$

Since these limits are approximate, it suffices to keep the log terms only

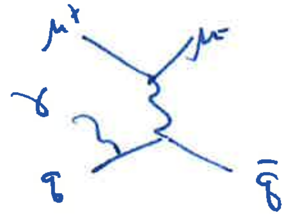
$$\text{Prob}(\to \gamma) \simeq \frac{\alpha\, Q_f^2}{\pi} \int_{p_T}^{M/2} \frac{dq_\perp}{q_\perp} \int_{2q_\perp/M}^{1} \frac{dz}{z} \cdot 2$$

$$\cong \frac{\alpha Q_f^2}{\pi} \int_{p_T}^{M/2} \frac{dq_\perp}{q_\perp} \left(-\log\left[\frac{2q_\perp}{M}\right]\right) \cdot 2$$

$$\cong \frac{\alpha Q_f^2}{\pi} \frac{1}{2}\left(-\log^2 \frac{2}{M}\cdot\frac{M}{2} + \log^2 \frac{2p_T}{M}\right) \cdot 2$$

$$\cong \frac{\alpha Q_f^2}{\pi} \left(\log^2 \frac{M}{2p_T}\right)$$

Actually, this is the probability of radiating a photon from one leg of the initial $q\bar{q}$ pair



To include both legs, multiply by 2

$Q_f^2 = \frac{4}{9}$ for u, $\frac{1}{9}$ for d. The $q\bar{q} \to \mu^+\mu^-$ annihilation cross section is also $\sim Q_f^2$, so, say roughly

$$\langle Q_f^2 \rangle \cong \frac{4 \cdot \frac{4}{9} + 1 \cdot \frac{1}{9}}{4+1} \cong 0.4$$

in all

$$\text{Prob}(\to \gamma) = 2 \langle Q_f^2 \rangle \frac{\alpha}{\pi} \log^2 \frac{M}{2p_T}$$

b.) Evaluate with $\alpha = \frac{1}{137}$, $\langle Q_f^2 \rangle = 0.4$

$M = 300$ $\quad p_T = 30$

$$\text{Prob} \cong 0.5\% \qquad (5 \times 10^{-3})$$

c.) $$\text{Prob}(\to g) = \underbrace{2}_{\text{both legs}} \cdot \frac{4}{3} \frac{\alpha_s}{\pi} \int_{p_T}^{M/2} \frac{dq_\perp}{q_\perp} \int_{2q_\perp/M}^{1} \frac{dz}{z} \cdot 2$$

$$= \frac{8}{3} \frac{\alpha_s}{\pi} \log^2 \frac{M}{2p_T}$$

d) Evaluating with $\alpha_s = 0.2$ $M = 300$ $p_T = 30$,

Prob $(\to g) \cong 0.44$ or $\underline{44}\ \%$