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Problem Set #2 — Solutions

1.) a.)

$BR(J/\psi \to \text{hadrons direct}) = 64.1\%$

$BR(J/\psi \to e^+e^- \text{ or } \mu^+\mu^-) = 11.9\%$

$BR(J/\psi \to \text{virtual photon} \to \text{hadrons}) = 13.50\%$

$BR(J/\psi \to \gamma + \text{hadrons}) = 8.8\%$

$98.3\% \ (\pm 2\%)$ ✓

the Total Width is 92.9 keV

then the partial widths are:

$\Gamma(J/\psi \to \text{hadrons direct})$ 59.5 keV

$\Gamma(J/\psi \to e^+e^- \text{ or } \mu^+\mu^-)$ 11.0 keV

$\Gamma(J/\psi \to \gamma^* \to \text{hadrons})$ 12.5 keV

$\Gamma(J/\psi \to \gamma + \text{hadrons})$ 8.2 keV b)

$BR(\psi' \to \text{hadrons direct})$	10.6 %
$BR(\psi' \to e^+e^- \text{ or } \mu^+\mu^-)$	1.58%
$BR(\psi' \to \gamma^* \to \text{hadrons})$	1.73%
$BR(\psi' \to \gamma + \text{hadrons})$	1.03%
$BR(\psi' \to \tau^+\tau^-)$	0.31%
$BR(\psi' \to J/\psi + (\pi^+\pi^-), (\pi^0\pi^0), \eta)$	56.07%
$BR(\psi' \to \chi_c + \gamma)$	28.65%
P 0^{++} 1^{++} 2^{++} states	
	99.97% ✓

the total width is 298 keV

the partial widths are:

$\Gamma(\psi' \to \text{hadron direct})$	31.7 keV
$\Gamma(\psi' \to e^+e^- \text{ or } \mu^+\mu^-)$	4.72
$\Gamma(\psi' \to \gamma^* \to \text{hadrons})$	5.17
$\Gamma(\psi' \to \gamma + \text{hadrons})$	3.08
$\Gamma(\psi' \to \tau^+\tau^-)$	0.93
$\Gamma(\psi' \to J/\psi + (\pi\pi \text{ or } \eta))$	167.65
$\Gamma(\psi' \to \chi_c + \gamma)$	85.66

c.) hadrons direct $\Gamma(\psi')/\Gamma(J/\psi) = 0.53$

e^+e^- or $\mu^+\mu^-$ $\Gamma(\psi')/\Gamma(J/\psi) = 0.43$

$\to \gamma^* \to$ hadrons $\Gamma(\psi')/\Gamma(J/\psi) = 0.41$

$\gamma +$ hadrons $\Gamma(\psi')/\Gamma(J/\psi) = 0.38$

The input BR's are very different, but the ratios of partial widths are consistent with

$$\frac{\Gamma(\psi')}{\Gamma(J/\psi)} \approx 0.4$$

for processes that involve $c\bar{c}$ annihilation. Annihilation depends on the wavefunction only through the value of

$$\psi(\vec{r}=0)$$

so we infer

$$\frac{|\psi(\vec{r}=0)|^2_{\psi'}}{|\psi(\vec{r}=0)|_{J/\psi}} \cong 0.4$$

The BR's are very different because of the addition of new decay processes

$$\psi' \to J/\psi + (\pi\pi, \eta) \qquad \psi' \to \gamma + \chi_c$$

2. a) For $\Delta(I=3/2) \longrightarrow \pi(I_1=1)\ N(I_2=\tfrac{1}{2})$

the decay matrix element should be proportional to

$$\langle I_1 I_2 I_1^3 I_2^3 | I_1 I_2\, I\, I^3 \rangle$$

then

$$\Gamma(\Delta(I^3) \to \pi(I_1^3) + N(I_2^3)) \propto |\langle I_1^3 I_2^3 | I I^3 \rangle|^2$$

the values of the squares of the Clebsch Gordan coefficients are

	Δ^{++}	Δ^{+}	Δ^{0}	Δ^{-}
$\pi^+ p$	1	0	0	0
$\pi^+ n$	0	1/3	0	0
$\pi^0 p$	0	2/3	0	0
$\pi^0 n$	0	0	2/3	0
$\pi^- p$	0	0	1/3	0
$\pi^- n$	0	0	0	1

so, since the relevant masses and other factors are essentially equal,

$\Delta^{++} \rightarrow \pi^+ p \qquad BR = 1$

$$\Delta^+ \rightarrow \left\{ \begin{array}{ll} \pi^+ n & BR = 1/3 \\ \pi^0 p & BR = 2/3 \end{array} \right.$$

$$\Delta^0 \rightarrow \left\{ \begin{array}{ll} \pi^0 n & BR = 2/3 \\ \pi^- p & BR = 1/3 \end{array} \right.$$

$\Delta^- \rightarrow \pi^- n \qquad BR = 1$

b.) For $N^* \ (I = 1/2) \rightarrow \pi N$, the relevant Clebsch Gordon coefficients are:

$I = 1/2 \; I^3 \rightarrow I_1 = 1 + I_2 = 1/2$

	N^{*+}	N^{*0}
$\pi^+ n$	2/3	0
$\pi^0 p$	1/3	0
$\pi^0 n$	0	1/3
$\pi^- p$	0	2/3

then

$N^{*+} \to \pi^+ n \quad BR = 2/3 \cdot 60\% = 40\%$

$\pi^0 p \quad BR = 1/3 \cdot 60\% = 20\%$

$N^{*0} \to \pi^0 n \quad BR = 1/3 \cdot 60\% = 20\%$

$\pi^- p \quad BR = 2/3 \cdot 60\% = 40\%$

c.) First, compute the BR's for $N^* \to \pi\Delta$. These are proportional to the squares of Clebsch Gordan coefficients

$I = \frac{1}{2}\ I^3 \longrightarrow I_1 = 1 \quad I_2 = 3/2$

	N^{*+}	N^{*0}
$\pi^- \Delta^{++}$	$\frac{1}{2}$	0
$\pi^0 \Delta^+$	$\frac{1}{3}$	0
$\pi^+ \Delta^0$	$\frac{1}{6}$	0
$\pi^- \Delta^+$	0	$\frac{1}{6}$
$\pi^0 \Delta^0$	0	$\frac{1}{3}$
$\pi^+ \Delta^-$	0	$\frac{1}{2}$

so

$N^{*+} \longrightarrow \pi^- \Delta^{++} \quad BR = \frac{1}{2} \cdot 40\% = 20\%$

$\pi^0 \Delta^+ \quad BR = \frac{1}{3} \cdot 40\% = 13\%$

$\pi^+ \Delta^0 \quad BR = \frac{1}{6} \cdot 40\% = 7\%$

$N^{*0} \longrightarrow \pi^- \Delta^+ \quad BR = \frac{1}{6} \cdot 40\% = 7\%$

$\pi^0 \Delta^0 \quad BR = \frac{1}{3} \cdot 40\% = 13\%$

$\pi^+ \Delta^- \quad BR = \frac{1}{2} \cdot 40\% = 20\%$

then

			BR
$N^{*+} \rightarrow$	$\pi^- \Delta^{++} \rightarrow$	$\pi^- \pi^+ p$	$20\% \cdot 1$
	$\pi^0 \Delta^+ \rightarrow$	$\pi^0 \pi^+ n$	$13.3\% \cdot \frac{1}{3} = 4.4\%$
		$\pi^0 \pi^0 p$	$13.3\% \cdot \frac{2}{3} = 8.9\%$
	$\pi^+ \Delta^0 \rightarrow$	$\pi^+ \pi^0 n$	$6.7\% \cdot \frac{2}{3} = 4.4\%$
		$\pi^+ \pi^- p$	$6.7\% \cdot \frac{1}{3} = 2.2\%$
$N^{*0} \rightarrow$	$\pi^- \Delta^+ \rightarrow$	$\pi^- \pi^+ n$	$6.7\% \cdot \frac{1}{3} = 2.2\%$
		$\pi^- \pi^0 p$	$6.7\% \cdot \frac{2}{3} = 4.4\%$
	$\pi^0 \Delta^0 \rightarrow$	$\pi^0 \pi^0 n$	$13.3\% \cdot \frac{2}{3} = 8.9\%$
		$\pi^0 \pi^- p$	$13.3\% \cdot \frac{1}{3} = 4.4\%$
	$\pi^+ \Delta^- \rightarrow$	$\pi^+ \pi^- n$	$20\% \cdot 1$

Add together the modes with identical final states – assuming no Q.M. interference. Then:

$N^{*+} \longrightarrow$	$\pi^+\pi^- p$	22.2%	
	$\pi^0\pi^0 p$	8.9%	40%
	$\pi^+\pi^0 n$	8.9%	

$N^{*0} \longrightarrow$	$\pi^+\pi^- n$	22.2%	
	$\pi^0\pi^0 n$	8.9%	40%
	$\pi^0\pi^- p$	8.9%	