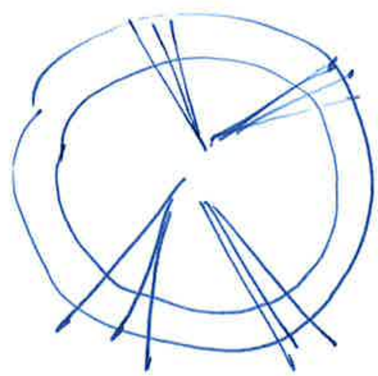
Physics 152 - 252

Final Exam — Solutions

1.) a) i.) both W's decay hadronically

e.g. $e^+e^- \rightarrow W^+ \quad W^-$
$\quad\hookrightarrow u\bar{d} \quad \hookrightarrow s\bar{c}$

the event appears as 4 jets of hadrons

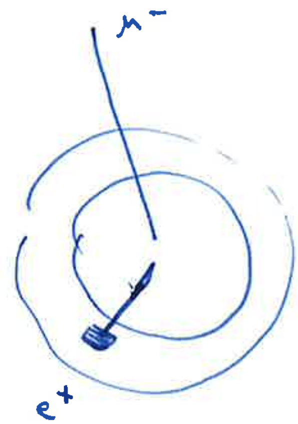


the jets are measured by the tracker and the EM and hadron calorimeters

ii.) both W's decay leptonically

e.g. $e^+e^- \rightarrow W^+ \quad W^-$
$\quad\hookrightarrow e^+\nu \quad \hookrightarrow \mu^-\bar{\nu}$

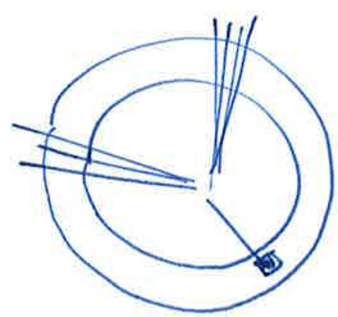
The neutrinos are not observed. An electron appears as a track plus energy in the EM calorimeter. A muon appears as a track that goes through the hadron calorimeter depositing minimal energy



iii) one W decays hadronically, one leptonically

e.g. $e^+e^- \to W^+ W^-$

$W^+ \to e^+\nu$, $W^- \to d\bar{u}$



The event appears as 2 jets plus an electron or muon.

The leptonic decay of a W can also be to a τ, observed as

$\tau \to e\nu\bar{\nu}$ $\qquad$ $\tau \to \pi\nu$

$\tau \to \mu\nu\bar{\nu}$ $\qquad$ $\tau \to \pi\pi^0\nu \to \pi\gamma\gamma\nu$

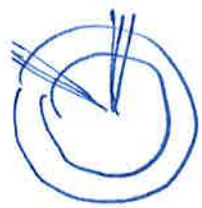
$\tau \to 3\pi\nu$

b.) Final states appearing in $e^+e^- \to Z Z$ that do not appear in $e^+e^- \to W^+W^-$:

$$e^+e^- \to Z \; Z$$
$$\quad \hookrightarrow \ell^+\ell^- \quad \hookrightarrow \ell^+\ell^-$$

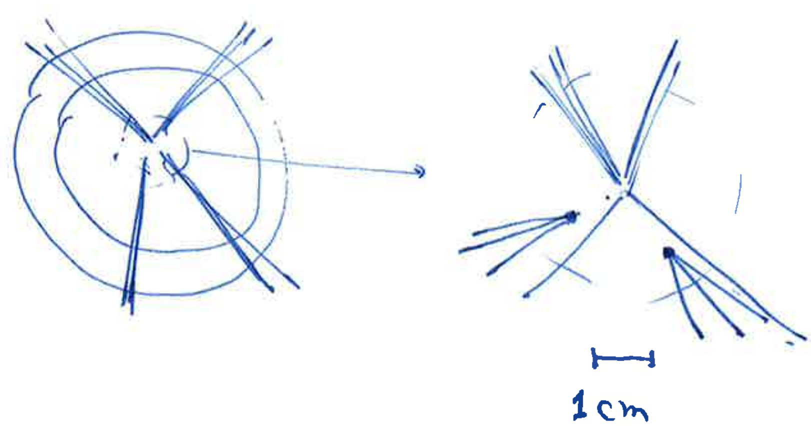
$$e^+e^- \to Z \qquad Z$$
$$\quad \hookrightarrow \underset{b\bar{b}}{q\bar{q}} \quad \hookrightarrow \nu\bar{\nu}$$

this appears as 2 jets, summing to the mass of the Z, plus nothing else



$$e^+e^- \to Z \qquad Z$$
$$\quad \hookrightarrow \underset{b\bar{b}}{q\bar{q}} \quad \hookrightarrow b\bar{b}$$

this is a 4-jet event with one or two jets having ~ 1cm displaced vertices from b decay



2.) a.)

$$\mathcal{M} = \frac{4G_F}{\sqrt{2}} \langle \pi^0 e^+ \nu | d_L^\dagger \bar{\sigma}^\mu u_L \, \nu_L^\dagger \bar{\sigma}_\mu e_L | \pi^+ \rangle$$

$$= \frac{4G_F}{\sqrt{2}} \langle \pi^0 | d_L^\dagger \bar{\sigma}^\mu u_L | \pi^+ \rangle \langle e^+ \nu | \nu_L^\dagger \bar{\sigma}_\mu e_L | 0 \rangle$$

b.) In the matrix element

$$\langle \pi^+(p) | j^{\mu 3} | \pi^+(p) \rangle$$

the right-hand side must be a 4-vector. So

$$\langle \pi^+(p) | j^{\mu 3} | \pi^+(p) \rangle = A p^\mu$$

since p^μ is the only 4-vector available. The left-hand side has dimension

$$(\text{GeV})^{-1} \cdot (\text{GeV})^3 \cdot (\text{GeV})^{-1} \sim \text{GeV}$$

so A is dimensionless.

$j^{\mu 3}$ carries zero momentum, so for $\mu = 0$ the matrix element is proportional to

$$\langle \int d^3x \, j^{03} \rangle = \langle I^3 \rangle$$

and

$$\langle \pi^+(p) | I^3 | \pi^+ \rangle = 1 \cdot \underbrace{\overbrace{2m_\pi}}_{}$$

from relativistic normalization

for a π^+ at rest, $p^\mu = (m_\pi, \vec{0})$, then

$$\langle \pi^+(p) | \cdot j^{\mu 3} | \pi^+(p) \rangle = 1 \cdot 2 m_\pi \, (1, \vec{0})^\mu$$

which agrees with (2)

c.) For a spin-1 multiplet

$$\langle m=1 | I^3 | m=1 \rangle = +1$$

$$\langle m=0 | I^- | m=1 \rangle = \sqrt{2} \qquad I^- = I^1 - i I^2$$

then

$$\langle \pi^0(p) | \cdot j^{\mu -} | \pi^+ \rangle = \langle \pi^0(p) | j^{\mu 1} - i j^{\mu 2} | \pi^+ \rangle$$

$$= 2 p^\mu \cdot \sqrt{2}$$

The relation between this and $d^\dagger \bar{\sigma}^\mu u$ is

for $Q = \begin{pmatrix} u \\ d \end{pmatrix}$ $\quad \sigma^- = \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix} = \dfrac{\sigma^1 - i\sigma^2}{2}$

$$d_L^\dagger \bar{\sigma}^\mu u_L = \bar{Q} \gamma^\mu \left(\frac{1-\gamma^5}{2}\right) \sigma^- Q = \frac{1}{2}\left(j^{\mu -} - j^{\mu 5 -}\right)$$

$j^{\mu 5 -}$ has the opposite parity for $j^{\mu -}$, so its matrix element $\langle \pi^0 | j^{\mu 5 -} | \pi^+ \rangle = 0$ by parity

Then

$$\langle \pi^0(p) | \, d_L^\dagger \bar\sigma^\mu u_L \, | \pi^+(p) \rangle$$

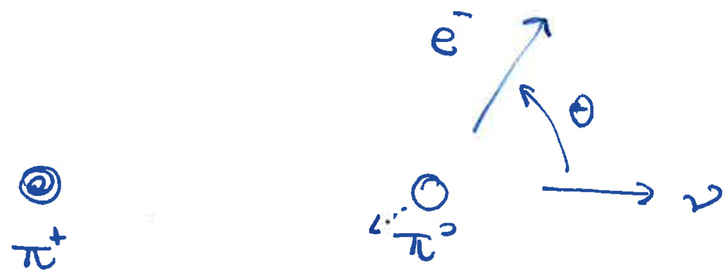
$$= \langle \pi^0 | \tfrac{1}{2} j^{\mu -} | \pi^+ \rangle = \tfrac{1}{2} \, 2p^\mu \cdot \sqrt{2}$$

in particular, for a π^+ at rest (and π^0 almost at rest)

$$\langle \pi^0 | \, d_L^\dagger \bar\sigma u_L | \pi^+ \rangle = \sqrt{2} \, (m_\pi, \vec{0})^\mu$$

d.) $$\langle e^+ \nu | \, \nu_L^\dagger \bar\sigma^\mu e_L | 0 \rangle = u^\dagger(\nu) \, \bar\sigma^\mu \, v(e)$$

Choose coordinates so that the ν goes out along the $\hat{3}$ axis



$$u(\nu) = \sqrt{2E_\nu} \begin{pmatrix} 0 \\ 1 \end{pmatrix} \qquad v(e) = \sqrt{2E_e} \begin{pmatrix} -\sin\theta/2 \\ \cos\theta/2 \end{pmatrix}$$

then

$$u^\dagger \bar\sigma^\mu v = \sqrt{4E_\nu E_e} \, (0 \; 1) \, (1, -\vec\sigma)^\mu \begin{pmatrix} -\sin\theta/2 \\ \cos\theta/2 \end{pmatrix}$$

$$= \sqrt{4E_\nu E_e} \, [\cos\theta/2 \, , \, x \, , \, x \, , \, x \,]^\mu$$

$$u^\dagger \bar\sigma^\mu v \cdot (m_\pi, 0)_\mu = m_\pi \sqrt{4E_\nu E_e} \, \cos\theta/2$$

e) $$\Gamma(\pi^+ \to \pi^0 e^+ \nu) = \frac{1}{2m_\pi} \int \frac{d^3p_\pi}{(2\pi)^3 2E_\pi} \int \frac{d^3p_\nu}{(2\pi)^3 2E_\nu} \int \frac{d^3p_e}{(2\pi)^3 2E_e}$$

$$\cdot (2\pi)^4 \delta(p_{\pi^+} - p_{\pi^0} - p_e - p_\nu) \; |\mathcal{M}|^2$$

with

$$|\mathcal{M}|^2 = 8G_F^2 \cdot (\sqrt{2})^2 \cdot m_\pi^2 \; 4E_\nu E_e \cos^2\theta/2$$

f.) $$\Gamma = \frac{1}{2m_\pi} \cdot \frac{1}{2E_{\pi^0}} \int \frac{d^3p_\nu}{(2\pi)^3 2E_\nu} \frac{d^3p_e}{(2\pi)^3 2E_e}$$

$$\cdot 2\pi \, \delta(\Delta m - E_e - E_\nu)$$

$$\cdot 16 G_F^2 \; m_\pi^2 \; 2E_\nu \, 2E_e \, \cos^2\theta/2$$

The recoiling π^0 absorbs the momentum of the ν and e^+. If $m_{\pi^0} \gg \Delta m$, $E_{\pi^0} \approx m_{\pi^0}$ in the final state

$$\Gamma = \frac{1}{2m_\pi} \cdot \frac{1}{2m_\pi} \cdot \int \frac{dE_\nu \, E_\nu^2 \cdot 4\pi}{(2\pi)^3} \int \frac{dE_e \, E_e^2 \, 2\pi \, d\cos\theta}{(2\pi)^3}$$

$$\cdot m_\pi^2 \cdot 16 G_F^2 \cos^2\theta/2$$

$$\cdot 2\pi \, \delta(\Delta m - E_e - E_\nu)$$

$$\int_{-1}^{1} d\cos\theta \;\; \cos^2\theta/2 \;=\; \int_{-1}^{1} d\cos\theta \cdot \tfrac{1}{2} \;=\; 1$$

so

$$I = \frac{32\, G_F^2\, \pi^2}{(2\pi)^5} \int_0^{\Delta m} dE_e \;\; E_e^2 \;\; (\Delta m - E_e)^2$$

$$= \frac{32\, G_F^2 \pi^2}{(2\pi)^5} (\Delta m)^5 \int_0^1 dx \;\; x^2 (1-x)^2$$

$$\int_0^1 dx \;\; x^2(1-x)^2 = \int_0^1 dx \,(x^2 - 2x^3 + x^4) = \frac{1}{3} - \frac{2}{4} + \frac{1}{5}$$

$$= \frac{1}{30}$$

$$I = \frac{G_F^2}{30\pi^3} (\Delta m)^5$$

g.) $$I = \frac{(1.166 \times 10^{-5}\ \mathrm{GeV}^{-2})^2\ (4.6 \times 10^{-3}\ \mathrm{GeV})^5}{30\pi^3}$$

$$= 3.0 \times 10^{-25}\ \mathrm{GeV}$$

$$= 3.0 \times 10^{-22}\ \mathrm{MeV}$$

h.) $\Gamma = 6.582 \times 10^{-22}$ MeV sec / 2.6×10^{-8} sec

$= 2.5 \times 10^{-14}$ MeV

so

$$BR(\pi^+ \to \pi^0 e^+ \nu) = 1.2 \times 10^{-8}$$

for comparison, the PDG value is 1.0×10^{-8}

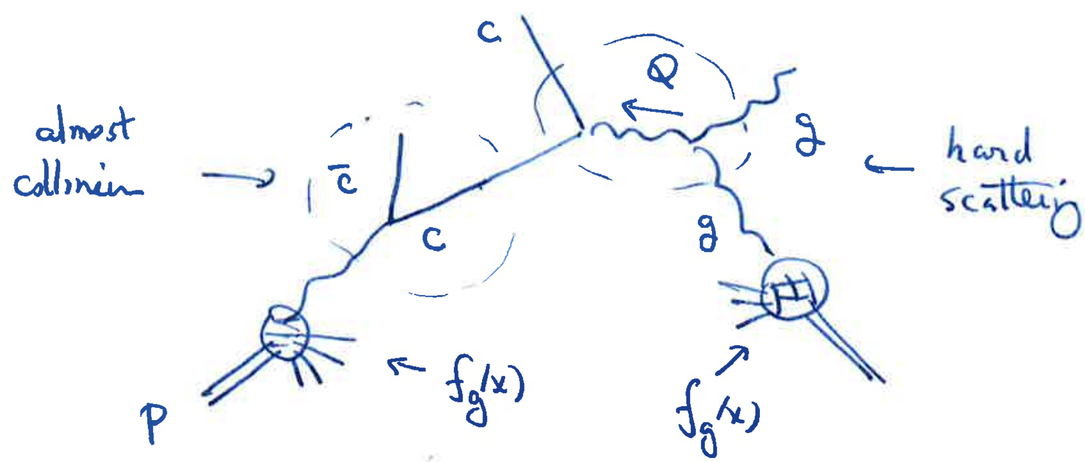
This includes the effect of the nonzero electron mass.

3.) a.) $\dfrac{\text{momentum in } g}{\text{momentum in } p} = \int_0^1 dx \; x \; f_g(x)$

here $\int_0^1 dx \; x \cdot \frac{3}{2}(1-x)^2/x = \frac{3}{2}\int_0^1 dx \, (1-x)^2 = \frac{3}{2} \cdot \frac{1}{3}$

so $\dfrac{\text{momentum in } g}{\text{momentum in } p} = \dfrac{1}{2}$

b.)



$\#$ of c quarks $= \int dy \, f_c(y)$

c.) $\int dy \, f_c(y) = \int dx \; f_g(x) \cdot \frac{\alpha_s}{\pi} \int \frac{dp_\perp}{p_\perp} \int dz \; P_{g \leftarrow c}(z)$

$= \int dx \; f_g(x) \; \frac{\alpha_s}{\pi} \int \frac{dp_\perp}{p_\perp} \int dz \; \frac{1}{2}\left(z^2 + (1-z)^2\right)$

d:) $$\int_0^1 dz\ \tfrac{1}{2}\left[z^2+(1-z)^2\right] = \tfrac{1}{2}\cdot\left(\tfrac{1}{3}+\tfrac{1}{3}\right) = \tfrac{1}{3}$$

e.) $$\int \frac{dp_\perp}{p_\perp} = ?$$

lower limit $p_\perp \sim m_c$ below this, cannot ignore the mass of c

upper limit $p_\perp \sim Q$ above this, the splitting is not approximately collinear

$$= \log\left(\frac{Q}{m_c}\right)$$

f.) $$\int dx\ f_g(x) = ?$$

upper limit $x \sim 1$ why not?

lower limit $x \sim Q^2/s$

so that $\hat{s} = x_1 x_2 s > Q^2$

$$\int_{Q^2/s}^1 dx\ \frac{3}{2}\frac{(1-x)^2}{x} = \frac{3}{2}\int_{Q^2/s}^1 dx\left(\frac{1}{x} - 2 + x\right)$$

$$\simeq \frac{3}{2}\left[\log\left(\frac{s}{Q^2}\right) - 2 + \frac{1}{2}\right]$$

$$\simeq \frac{3}{2}\left\{2\log\frac{E_{cm}}{Q} - \frac{3}{2}\right\}$$

then

$$\int dy\, f_c(y) \cong \frac{1}{3} \cdot \frac{\alpha_s}{\pi} \cdot \log \frac{Q}{m_c} \cdot \left(3 \log \frac{E_{cm}}{Q} - \frac{9}{4} \right)$$

$$\cong \frac{\alpha_s}{\pi} \log\left(\frac{Q}{m_c}\right) \left(\log \frac{E_{cm}}{Q} - \frac{3}{4}\right)$$

h.) Numerically, with

$E_{cm} = 13{,}000$ GeV

$Q = 200$ GeV $\qquad\qquad \alpha_s = 0.118$

$m_c = 1$ GeV

$$\int dy\, f_c(y) \cong 0.7$$

so, at large momentum transfer, the number of c quarks in the proton is $\mathcal{O}(1)$!