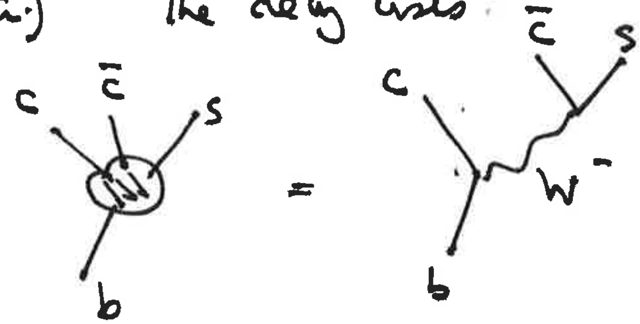
Physics 152 / 252

Final Exam – Solutions

1.) a.) The decay uses



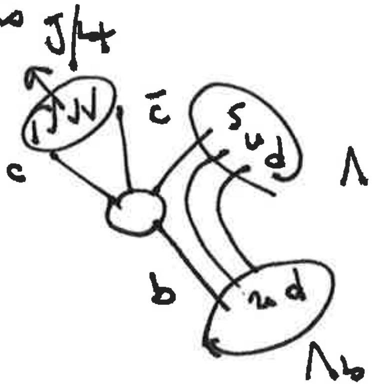
with the elementary

$b \to c$

$c \to s$

weak interaction transitions

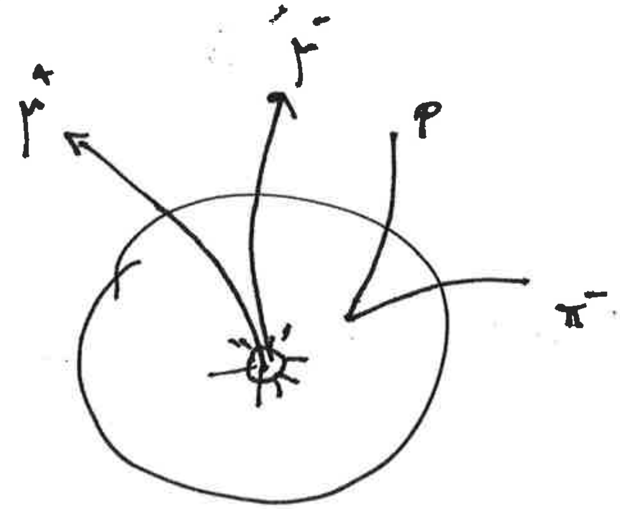
as follows



b.) The ultimate final state of the decay is $\mu^+\mu^- p\pi^-$

All four of the final-state particles are charged, so their momenta are measured in the tracker, that is, a device that can locate points along a charged particle track in a solenoidal magnetic field.

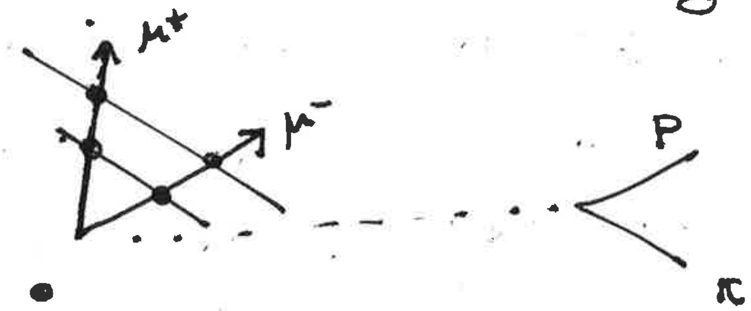
The Λ has a macroscopic lifetime ($c\tau = 8cm$) so the p and π^- tracks originate from a point well displaced from the primary interaction. Muons are identified as isolated tracks that penetrate meters of absorber. Looking down the beam axis, the Λ_b^0 appears as:



c.) $$c\tau = 1.45 \times 10^{-12} \text{sec} \cdot c$$

$$= 0.43 \text{ mm}$$

d) Modern collider detectors have silicon sensors located a few cm from the collision point. These detect space points on the track to an accuracy below 100μ. This allows one to recognize that the $\mu^+\mu^-$ pair does not originate from the primary vertex but, rather, from a finite distance away



The distribution of decay distances ℓ is

$$e^{-\ell/c\tau\gamma}$$

where $\gamma = E_\Lambda/m_\Lambda$. γ can be computed from the four measured final-state 4-vectors.

e.) The decay $\Lambda \to n\pi^0$ does not produce any tracks. The π^0 is observed as 2 photons in the electromagnetic calorimeter, and the n is simply a hit in the hadron calorimeter. Thus, it is difficult to recognize the Λ in this event.

2.a) The initial 4-vectors are

$$p_e = (m_e,\ 0\,0\,0)$$

$$p_{\bar\nu} = (E,\ 0\,0\,E)$$

then $m_W^2 = (p_e + p_\nu)^2 = (E+m_e)^2 - E^2 \cong 2E\,m_e$

$$E = \frac{m_W^2}{2m_e} = 6.3 \times 10^6 \text{ GeV}$$

$$= 6.3 \text{ PeV}$$

b.) A $\bar\nu_e$ produced in a weak decay is <u>right</u> handed $S^3 = +\frac{1}{2}$

c.) The process is

$$\mathcal{M} = \frac{g}{\sqrt{2}}\, v_L^\dagger\, \bar\sigma^\mu\, u_L\, \varepsilon_\mu^*(W)$$

with

$$u_L = \sqrt{m_e}\begin{pmatrix}1\\0\end{pmatrix} \qquad u_L = \sqrt{m_e}\begin{pmatrix}0\\1\end{pmatrix}$$

for the two possible e^- polarizations

and

$$v_L = \sqrt{2E_\nu}\begin{pmatrix}0\\1\end{pmatrix}$$

the spinor is <u>opposite</u> to the spin of the $\bar{\nu}_e$

For $u_{eL} = \sqrt{m_e}\begin{pmatrix}0\\1\end{pmatrix}$

$$v_L^\dagger \bar{\sigma}^\mu u_L = \sqrt{2E_\nu m_e}\;(0\;1)\,(1,-\vec{\sigma})\begin{pmatrix}0\\1\end{pmatrix}$$

$$= \sqrt{2E_\nu m_e}\;(1\;0\;0\;1)^\mu$$

To a very good approximation, this vector is parallel to the W momentum vector $p_W = \left(E_W,\; 0\; 0\; E_W - \underbrace{\frac{m_W^2}{2E_W}}_{\text{tiny!}}\right)$

Since $\epsilon_W^* \cdot p_W = 0$ this state gives essentially no contribution

For $u_{eL} = \sqrt{m_e}\begin{pmatrix}1\\0\end{pmatrix}$

$$v_L^\dagger \bar{\sigma}^\mu u_L = \sqrt{2E_\nu m_e}\;(0\;1)\,(1,-\vec{\sigma})^\mu\begin{pmatrix}1\\0\end{pmatrix}$$

$$= \sqrt{m_W^2}\;(0,\,-1,\,-i,\,0)^\mu$$

$$= -m_W\cdot\sqrt{2}\cdot\frac{1}{\sqrt{2}}\,(0,\,1,\,i,\,0)^\mu$$

so this reaction produces a W with $S^3 = +1$, as we would expect

d.) Then $|\mathcal{M}(\nu\, e^-(R))|^2 = g^2/2 \cdot 2m_W^2 = g^2 m_W^2$

$$|\mathcal{M}(\nu\, e^-(L))|^2 = 0$$

$$\sigma = \underbrace{\frac{1}{2}}_{e^-\text{ pol. avg.}} \frac{1}{2E_\nu\, 2m_e \cdot \underbrace{1}_{|v_\nu - v_e|}} \int d\Pi_1\, |\mathcal{M}|^2$$

$$= \frac{1}{4}\frac{1}{m_W^2}\, 2\pi\, \delta(s - m_W^2)\, g^2 m_W^2$$

$$= 2\pi^2 \frac{g^2}{4\pi}\, \delta(s - m_W^2) = 2\pi^2 \alpha_w\, \delta(s - m_W^2)$$

e.) $A = 2\pi^2 \alpha_w = 0.66$

f.) For each electron, the rate of W^- production is

$$\text{events/sec} = \int dE \cdot \frac{1}{9} \cdot \frac{d\Phi}{dE}\, \sigma(E)$$

we need to convert

$$\delta(s - m_W^2) = \delta(2E_\nu m_e - m_W^2)$$

$$= \frac{1}{2m_e}\, \delta(E_\nu - 6.3\,\text{PeV})$$

then

$$\text{events/sec} = \frac{1}{9} \left.\frac{d\Phi}{dE}\right|_{E=6.3\,\text{PeV}} \cdot 0.66\, \frac{1}{2m_e}$$

evaluate this numerically

$$\text{events/sec} = \frac{1}{9}\,10^{-7}\,\frac{1}{(E_\nu(GeV))^2}\,\frac{0.33}{(m_e(GeV))}\,\frac{1}{(GeV)^2 cm^2\,sec}$$

$$\frac{1}{(GeV)^2 cm^2} = \frac{10^{-27}}{GeV^2\,mb} = 0.39\times10^{-27}$$

$$\text{events/sec} = \frac{1}{9}\,10^{-7}\,\frac{1}{(6.3\times10^{6})^2}\,\frac{0.33}{(0.51\times10^{-3})}\,0.39\times10^{-27}$$

$$= 7.1\times10^{-47}/sec$$

g.) Each electron comes with a mass of

$$2 \text{ nucleons} = 2\times1.7\times10^{-24}\,g$$

so an event rate of 1/yr requires a mass of

$$(7.1\times10^{-47}/sec)^{-1}(3.16\times10^{7}\,sec/yr)^{-1}(3.3\times10^{-24}\,g/e^-)$$

$$= 1.5\times10^{15}\,g$$

For 5 events per year from $W^-\to\mu\nu$ or $W^-\to\tau\nu$ we would need

$$1.5\times10^{15}\,g \times 5 \times \frac{1}{BR(W\to\mu\text{ or }\tau)}$$

since $BR(W\to\mu) \cong BR(W\to\tau) \cong 11\%$

$$\cong \frac{1}{3+2\cdot 3} \qquad \frac{1}{e\,\mu\,\tau\;(ud)\cdot(cs)}$$

this works out to

$$3.4\times10^{16}\ g\ \ H_2O$$

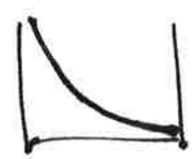
Water is conveniently $1\ g/cm^3$. So this is a cube of ice of volume

$$34\ km^3 = (3\ km)^3$$

This is large but not totally impractical; IceCube is $(1\ km)^3$ and will be active for many years.

h) In the W frame the W_R^- decays to a μ^- (which must be L-handed) with the angular distribution

$$\frac{d\Gamma}{d\cos\theta} \sim (1-\cos\theta)^2$$



The energy of the muon in the lab frame is found by boosting

$$P_\mu = \left(\frac{m_W}{2}, \frac{m_W}{2}\sin\Theta, 0, \frac{m_W}{2}\cos\Theta \right)$$

which boost to
$$E_\mu^{lab} = \left(\gamma \frac{m_W}{2} + \gamma\beta \frac{m_W}{2}\cos\Theta \right)$$

$$\gamma = \frac{E_W}{m_W} = 8 \times 10^4 \quad \text{so} \quad \beta \approx 1$$

$$E_\mu^{lab} = E_W^{lab} \cdot \frac{1}{2}(1+\cos\Theta_{cm})$$

Let
$$X = \frac{E_\mu^{lab}}{E_W^{lab}} \qquad X = \frac{1}{2}(1+\cos\Theta_{cm})$$
$$1-X = \frac{1}{2}(1-\cos\Theta_{cm})$$

and so
$$\frac{d\Gamma}{dx} \sim (1-x)^2$$

3.) a.) The probability to emit a collinear gluon in the process of $q\bar{q}$ annihilation is

$$\int dz \int \frac{dp_T^2}{p_T^2} \cdot \frac{\alpha_s}{2\pi} P_{g\leftarrow p}(z) \cdot \underbrace{2}_{\text{either } q \text{ or } \bar{q} \text{ can emit}}$$

$$= \int dz \int \frac{dp_T^2}{p_T^2} \cdot \frac{4}{3} \frac{1+(1-z)^2}{z} \cdot \frac{\alpha_s}{2\pi} \cdot 2$$

$$\sim \int \frac{dz}{z} \int \frac{dp_T}{p_T} \frac{4}{3} \cdot 2 \cdot 2 \quad \frac{\alpha_s}{\pi}$$

both logs can be estimated as $\sim \log\left(\frac{400\ \text{GeV}}{50\ \text{GeV}}\right)$

$$\sim \frac{16}{3} \cdot \frac{\alpha_s}{\pi} \cdot \log^2\left(\frac{400}{50}\right)$$

Prob ~ 90% ie almost certain

b.) For photons the probability is . . .

$$\int dz \int \frac{dp_T^2}{p_T^2} \cdot \frac{\alpha}{2\pi} \cdot Q_f^2 \cdot \frac{1+(1-z)^2}{z} \cdot 2$$

$$\sim \int \frac{dz}{z} \int \frac{dp_T}{p_T} \frac{\alpha}{\pi} \cdot 4 \cdot \langle Q_f^2 \rangle$$

If the annihilations are $2/3$ $u\bar{u}$, $\frac{1}{3}$ $d\bar{d}$, as expected for valence quarks in the proton

$$\langle Q_f^2 \rangle = \frac{2}{3} \cdot \frac{4}{9} + \frac{1}{3} \cdot \frac{1}{9} = \frac{7}{27} \cong 0.26$$

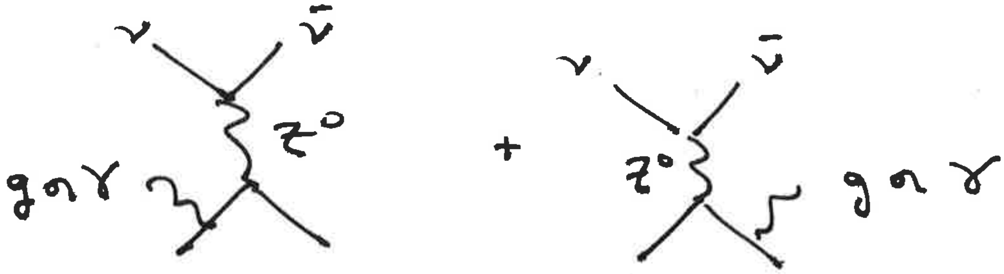
$$\text{Prob.} \sim 4 \cdot (0.26) \cdot \frac{\alpha}{\pi} \cdot \left(\log^2 \frac{400}{50}\right)$$

$$\sim 1\%$$

c.) A Z^0 produced in the Drell-Yan process can decay to $\nu\bar{\nu}$ (20% of the time).

If a gluon or photon is radiated from the initial state we have

$$q\bar{q} \rightarrow (g \text{ or } \gamma) + \nu\bar{\nu}$$



This is essentially indistinguishable from

$$q\bar{q} \to (g \text{ or } \gamma) + (\chi\bar{\chi})$$

The rate of this process must be carefully evaluated and subtracted from the observed number of events.