Physics 152/252

Final Exam : Solutions

1.) a.) W^+ has the possible decays:

$W^+ \to$	$e^+\nu$	$\mu^+\nu$	$\tau\nu$	$u\bar{d}$	$c\bar{s}$
relative ratio	1	1	1	3	3

$\propto$ # color states

(ignoring QCD corrections)

W^- has the same decay pattern

so

$t\bar{t} \to b\bar{b} + \ell^+\ell^- + \nu\bar{\nu}$ $\quad 3\cdot 3 = 9$

$t\bar{t} \to b + \ell^+\nu + \bar{b} + q\bar{q}$ $\quad 3\cdot 6 = 18$

$t\bar{t} \to b + q\bar{q} + \bar{b} + \ell^-\nu$ $\quad 3\cdot 6 = 18$

$t\bar{t} \to b + q\bar{q} + \bar{b} + q\bar{q}$ $\quad 6\cdot 6 = 36$

$\overline{81}$

so

$b\bar{b}\ell^+\ell^-$	$b\bar{b}\, q\bar{q}\, \ell\nu$	$b\bar{b} + 2\times q\bar{q}$
$1/9 = 11\%$	$4/9 = 44\%$	$4/9 = 44\%$

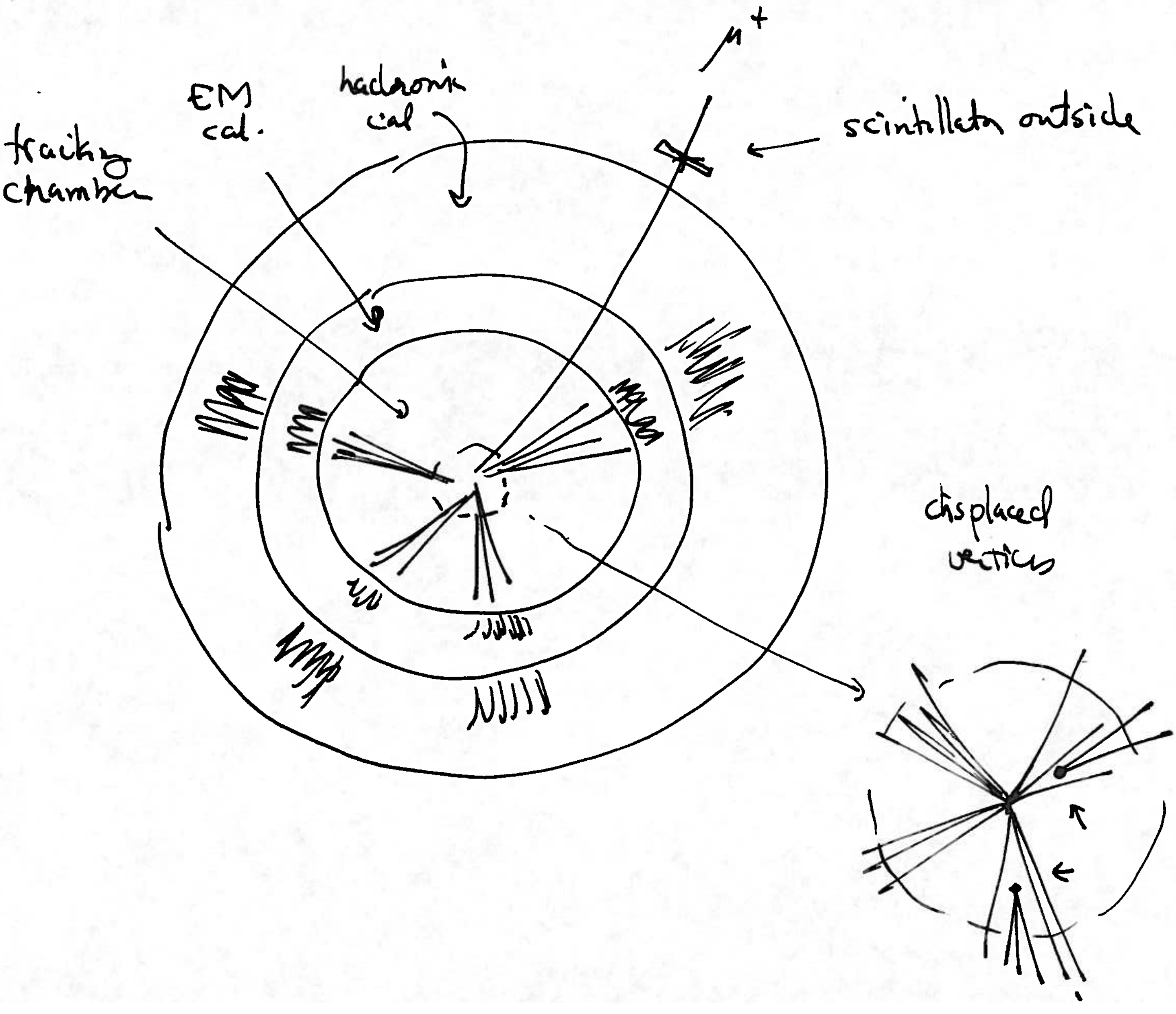
b.) This event would be: $t\bar{t} \to b\,\mu^+\nu_\mu\;\bar{b}\;d\bar{u}$

μ^+: detected by tracking – minimum ionizing particle

$b\,\bar{b}\,d\,\bar{u}$ detected as "jets" – tracks plus calorimetric energy deposition

$b\bar{b}$ detected as tracks originating from a displaced vertex

ν_μ not detected



2.) a.) Isospin is conserved. ψ' and J/ψ have $I=0$, so the pion isospin indices must combine to an invariant: δ^{ab}

With this structure, the decay branching ratios are

$$\underbrace{\pi^1\pi^1 \qquad \pi^2\pi^2}_{\pi^+\pi^-} \qquad \underbrace{\pi^3\pi^3}_{\pi^0\pi^0}$$

$$2/3 \quad : \quad 1/3$$

b.) For $m_\pi = 0$, the pion is a Goldstone boson. A Goldstone boson at $\vec{p}=0$ is a rotation of the vacuum $u\bar{u}$ and $d\bar{d}$ condensates. But the ψ' and J/ψ are $c\bar{c}$ bound states with no u, d content. So they do not care about the orientation of this condensate.

c.) $\psi' \to \psi\pi^0$ violates isospin symmetry. So we expect

$$\mathcal{M} \sim \mathcal{O}(\alpha) \text{ or } \mathcal{O}\left(\frac{m_d - m_u}{500\,\text{MeV}}\right) \sim 10^{-2}$$

$BR \sim 10^{-4}$ [PDG tables:

$BR(\psi' \to \psi\pi^0) = 1.3 \times 10^{-3}$]

d.) $$\Gamma(\psi' \to \psi \pi\pi) = \frac{1}{2m_{\psi'}} \int \frac{d^3p_\psi\, d^3p_{\pi 1}\, d^3p_{\pi 2}}{(2\pi)^9\, 2E_\psi\, 2E_{\pi 1}\, 2E_{\pi 2}}$$

$$\cdot (2\pi)^4 \delta^{(4)}(P_{\psi'} - \Sigma p)\; |A|^2 (p_1 \cdot p_2)^2\, \delta^{ab}\delta^{ab} \cdot \frac{1}{2}$$

$\mathcal{M}$ is independent of the spin of the ψ', J/ψ so there is no spin factor. The J/ψ receives the spin of the ψ'!

The final factor of $\frac{1}{2}$ accounts Bose statistics: $|\pi(p_1)\pi(p_2)\rangle$ and $|\pi(p_2)\pi(p_1)\rangle$ are identical states, but they are counted twice in the integral.

If the J/ψ and ψ' are heavy, we can integrate

$$\int \frac{d^3p_\psi}{(2\pi)^3} (2\pi)^3 \delta^{(3)}(\vec{p}_{\psi'} - \vec{p}_\psi - \vec{p}_{\pi 1} - \vec{p}_{\pi 2}) = 1$$

and set

$$E(\pi_1) + E(\pi_2) = m(\psi') - m(J/\psi) = \Delta m$$

in the energy-conserving delta-function. Then

$$\Gamma = \frac{1}{2m_\psi} \frac{1}{2m_\psi} \int \frac{d^3p_1\, d^3p_2}{(2\pi)^6\, 2E_1\, 2E_2}\; 2\pi\, \delta(\Delta m - E_1 - E_2)$$

$$\cdot\; |A|^2 \cdot (p_1 p_2)^2 \cdot \frac{3}{2}$$

also, for massless pions

$$E_1 = p_1 \qquad E_2 = p_2 \qquad p_1 \cdot p_2 = p_1 p_2 (1 - \cos\theta_{12})$$

$$I = \frac{1}{4m_\psi^2} \frac{1}{(2\pi)^5} \frac{1}{4} \; 4\pi \cdot 2\pi \int d\cos\theta_{12} \; (1-\cos\theta_{12})^2$$

$$\int dp_1 \, dp_2 \; \delta(\Delta m - p_1 - p_2) \; \frac{p_1^2 p_2^2}{p_1 p_2} (p_1 p_2)^2$$

$$= \frac{1}{8m_\psi^2} \frac{1}{8\pi^3} \int_{-1}^{1} d\cos\theta_{12} \; (1-\cos\theta_{12})^2$$

$$\cdot \int dp_1 \, dp_2 \; \delta(\Delta m - p_1 - p_2) \, (p_1 p_2)^3 \qquad |A|^2 \cdot \frac{3}{2}$$

The second line is $\sim (\Delta m)^7$ so

$$I \sim c \, \frac{|A|^2 (\Delta m)^7}{m_\psi^2}$$

c.) $$\int_{-1}^{1} d\cos\theta_{12} \; (1-\cos\theta)^2 = \int_0^2 dy \; y^3 = \frac{8}{3}$$

$$\int dp_1 \, dp_2 \; \delta(\Delta m - p_1 - p_2) \; p_1^3 p_2^3 =$$

$$= \int_0^{\Delta m} dp_1 \; p_1^3 \, p_2^3 = (\Delta m)^7 \int_0^1 dx \; x^3 (1-x)^3$$

$$= (\Delta m)^7 \int_0^1 dx \; x^3 (1 - 3x + 3x^2 - 1)$$

$$= (\Delta m)^7 \left[\frac{1}{4} - \frac{3}{5} + \frac{3}{6} - \frac{1}{7} \right] = \frac{(\Delta m)^7}{140}$$

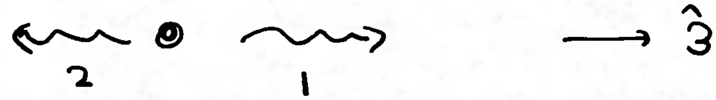
In all

$$I = \frac{3}{128\pi^3} \cdot \frac{8}{3} \frac{|A|^2}{m_4^2} (\Delta m)^7 \frac{1}{140}$$

$$= \frac{1}{2240\pi^3} \frac{|A|^2}{m_4^2} (\Delta m)^7$$

3.) a.) The vectors ε_1 and ε_2 must be orthogonal.

For $p_1, p_2 \parallel \hat{z}$



Eg.

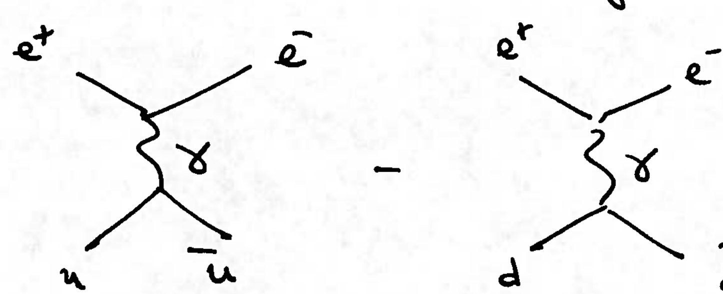
$\vec{\varepsilon}_1 = \hat{x} \qquad \vec{\varepsilon}_2 = \hat{y}$

or

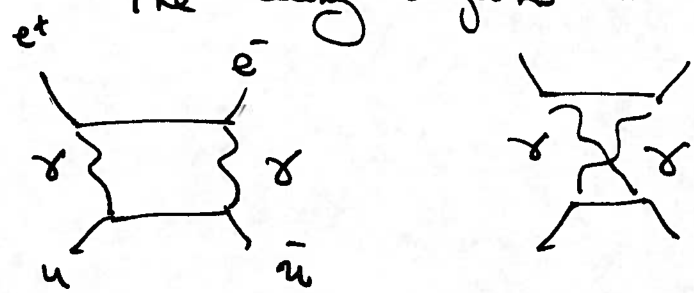
$\vec{\varepsilon}_1 = \frac{1}{\sqrt{2}}(x+iy) \qquad \vec{\varepsilon}_2 = \frac{1}{\sqrt{2}}(x-iy)$

b.) The structure $\varepsilon_1^* \cdot \varepsilon_2^*$ is invariant under parity but the π^0 has $P = -1$.

c.) $\pi^0 \to e^+e^-$ has an order e^2 diagram



but this must be zero, because it is $C = -1$ while π^0 is $C = +1$. The leading diagrams are



+ ($u \leftrightarrow d$)

Also, since π^0 has $J=0$, it must decay to

$$e^-_L e^+_L \,,\quad e^-_R e^+_R$$

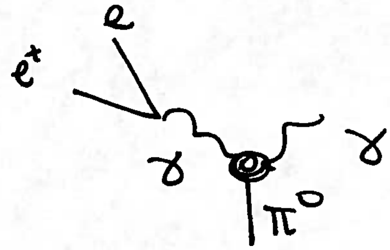
which requires a helicity flip (as in $\pi^+ \to e^+ \nu$).

So

$$BR(\pi^0 \to e^+ e^-) \sim (\alpha)\left(\frac{m_e}{m_{\pi^0}}\right)^2 = 10^{-7}$$

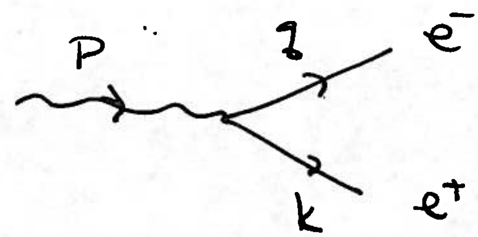
[PDG tables: $BR = 7\times 10^{-8}$]

d.)



e.) Put $\gamma \parallel \hat{3}$

in the collinear region:



$$P \cong (p, 0\ 0\ p)$$

$$q = \left(zp,\ p_T,\ 0,\ zp - \frac{p_T^2}{2zp}\right)$$

$$k = \left((1-z)p,\ -p_T,\ 0,\ (1-z)p - \frac{p_T^2}{2(1-z)p}\right)$$

more exactly $P = \left(p,\ 0,\ 0,\ p - \frac{p_T^2}{2z(1-z)p}\right)$

then $$p^2 = \frac{p_T^2}{z(1-z)}$$

for polarization, we need

$$\varepsilon_L^\mu(p) = \frac{1}{\sqrt{2}}\,(0\;1\;,-i\;,0)$$

$$u_L(q) = \sqrt{2zp}\,\begin{pmatrix} -p_T/2zp \\ 1 \end{pmatrix}$$

$$v_L(k) = \sqrt{2(1-z)p}\,\begin{pmatrix} +p_T/2(1-z)p \\ 1 \end{pmatrix}$$

then

$$u^\dagger \bar{\sigma}\cdot\varepsilon_L v = \sqrt{z(1-z)}\cdot 2p\cdot\frac{1}{\sqrt{2}}\left(-\frac{p_T}{2zp},1\right)\Big(\underbrace{\sigma^1 - i\sigma^2}_{2\sigma^-}\Big)\begin{pmatrix} p_T/2(1-z)p \\ 1 \end{pmatrix}$$

$$= \sqrt{z(1-z)}\;\sqrt{2}\;p\cdot\frac{p_T}{2(1-z)p}\cdot 2$$

$$= \sqrt{2}\;p_T\;\sqrt{\frac{z}{1-z}}$$

then

$$\mathcal{M}(\gamma_L \to e_L^- e_R^+) = \sqrt{2}\,e\,p_T\,\sqrt{\frac{z}{1-z}}$$

f) Reversing $e^- \leftrightarrow e^+$ (a symmetry by C)

$$\mathcal{M}(\gamma_L \to e_R^- e_L^+) = \sqrt{2}\;e\,p_T\,\sqrt{\frac{1-z}{z}}$$

Reversing all spins (a symmetry, by P)

$$\mathcal{M}(\gamma_R \to e^-_R e^+_L) = \sqrt{2}\, e\, p_T \sqrt{\frac{z}{1-z}}$$

$$\mathcal{M}(\gamma_R \to e^-_L e^+_R) = \sqrt{2}\, e\, p_T \sqrt{\frac{1-z}{z}}$$

g.) $$\Gamma(\pi^0 \to \gamma e^+ e^-) = \frac{1}{2m_\pi} \int \frac{d^3 p_\gamma}{(2\pi)^3 2E_\gamma} \int \frac{d^3 k\, d^3 q}{(2\pi)^6 2k\, 2q}$$

$$\cdot (2\pi)^4 \delta(p_{\pi^0} - \Sigma p) \left| \mathcal{M}(\pi^0 \to \gamma\gamma) \frac{1}{p^2} \mathcal{M}(\gamma \to e^+ e^-) \right|^2 \cdot 2$$

write $\dfrac{d^3 k}{(2\pi)^3} = \dfrac{d^3 p}{(2\pi)^3}$ $\quad d^3 q = p\, dz\, d^2 p_T = dz\, p\, 2\pi\, dp_T\, p_T$

$$2k \cdot 2q = 2p \cdot 2\, z(1-z)$$

The last factor of 2 is because either photon in $\pi^0 \to \gamma\gamma$ can split.

$$\Gamma(\pi^0 \to \gamma e^+ e^-) = \Gamma(\pi^0 \to \gamma\gamma) \cdot 2$$

$$\cdot \int \frac{dz\, p\, p_T\, dp_T\, 2\pi}{(2\pi)^3\, 2p\, z(1-z)} \left(\frac{1}{p^2}\right)^2$$

$$\cdot \sum_{L,R} |\mathcal{M}(\gamma \to e^-_L e^+)|^2$$

$$= \Gamma(\pi^0 \to \gamma\gamma) \cdot 2 \cdot \int \frac{dz}{8\pi^2} \frac{dp_T\, p_T}{z(1-z)} \left(\frac{z(1-z)}{p_T^2}\right)^2$$

$$\cdot 2 p_T^2 e^2 \left[\frac{z}{1-z} + \frac{1-z}{z}\right]$$

$$\Gamma(\pi^0 \to \gamma e^+ e^-) = \Gamma(\pi^0 \to \gamma\gamma)$$

$$\cdot\ 2 \cdot \frac{\alpha}{\pi} \cdot \int_0^1 dz \int \frac{dp_T}{p_T} \left[z^2 + (1-z)^2 \right]$$

Notice that the integral over p_T goes down to 0 if $m_e = 0$

h.) For m_e finite, the singularity $\left(\frac{1}{p^2}\right)^2$ is rounded off at $p^2 \sim p_T^2 \sim m_e^2$. Then

$$\int \frac{dp_T}{p_T} \approx \log \frac{m_\pi}{m_e}$$

also

$$\int_0^1 dz \left[z^2 + (1-z)^2 \right] = \frac{2}{3}$$

so

$$\frac{\Gamma(\pi^0 \to \gamma e^+ e^-)}{\Gamma(\pi^0 \to \gamma\gamma)} \simeq \frac{4}{3} \frac{\alpha}{\pi} \log \frac{m_\pi}{m_e}$$

i) Numerically

$$BR(\pi^0 \to \gamma e^+ e^-) = 1.7 \times 10^{-2}$$

$$\left[\text{PDG tables:} \quad BR = 1.2 \times 10^{-2} \right]$$