Physics 134 – Problem Set # 7

(due Tuesday, June 5)

1. In large atom, for example, iron or lead, the 1S states have roughly the shape of the Hydrogenic 1S wavefunctions about a point charge equal to the full nuclear charge, $Z = 26$ or $Z = 82$ in these two cases. The outer wavefunctions, on the other hand, see an almost completely screened charge. Then the overall size of the atom remains about 1 Å, while the size of the 1S wavefunction gets increasingly smaller, with $r \sim Z^{-1}$ Å. We can use this idea to study the photoelectric cross section in large atoms.

   (a) How does the energy to eject a 1S electron depend on Z? Estimate this energy for iron and lead.

   (b) How does the maximum value of the photoelectric cross section depend on Z?

   (c) Here is a crude model of a heavy atom in a complex molecule, for example, iron in hemoglobin: Model the 1S state as the appropriately scaled 1S state of Hydrogen, but model the outer wavefunctions as wavefunctions in an attractive spherical potential well of radius $R = 2$ Å and depth $\mathcal{E} = 5$ eV. Compute the photoelectric cross section as a function of energy for energies above but very close to the threshold. Graph the cross section over the region from the threshold to energies of 10 eV above threshold. How does the parameter R enter the shape of this curve?

2. Compute some rates for M1 transitions in Hydrogen and positronium.

   (a) The splitting of the $J = 0$ and $J = 1$ 1S states of Hydrogen is

   $$\frac{8}{3} g_p \alpha^2 \frac{m_e}{m_p} Ry$$

   Numerically, this is 5.86×10^{-6} eV, corresponding to a wavelength of 21 cm. The decay of the $J = 1$ state is an M1 transition dominated by the amplitude containing the electron magnetic moment. Compute the decay rate and lifetime of the $J = 1$ state.

   (b) The splitting of the $J = 0$ and $J = 1$ 1S states of positronium is

   $$\frac{7}{6} \alpha^2 Ry$$

   Numerically, this is 8.45×10^{-4} eV. The decay of the $J = 1$ state is an M1 transition with contributions from both the spin of the electron and the spin of the positron. Compute the decay rate and lifetime of the $J = 1$ state.