Physics 134 – Problem Set # 6

(due Thursday, May 24)

1. Compute the lifetime of the 3D states of Hydrogen.

   (a) Write out the electric dipole matrix element for one of the 3D states to decay to the 1S state, and show that this matrix element is zero. In the same way, show that the electric dipole matrix element of the 3D state to decay to the 2S state is zero. Then the only options for a leading-order decay of the 3D states are to the 2P states.

   (b) Ignoring electron spin, compute the dipole matrix element for the various 3D states to decay to the 2P states. Express the matrix element as the product of a radial contribution and an angular contribution. The radial matrix element should be the same in all cases. The angular matrix elements should be proportional to the Clebsch-Gordan coefficients for coupling $1 + 1 \to 2$; you can use this to check your work. Square the matrix elements and compute the decay rates. You can integrate over all possible photon directions; this simplifies the calculation. Show explicitly that all of the 3D states have equal decay rates.

   (c) Now add back the electron spin, and consider the effect of fine structure. Compute the matrix elements for the decay of the $J = 3/2$ and $J = 5/2$ 3D states to the $J = 3/2$ and $J = 1/2$ 2P states. Note that these are linear combinations of the matrix elements computed in part (b). Square and compute the total decay rates. Show that the $J = 5/2$ states decay only to $J = 3/2$ states, but that $J = 3/2$ states can decay both to $J = 3/2$ and $J = 1/2$ states. Show that, if you ignore the fact that the energies of the photons are changed slightly by the fine structure splitting, all states have the same total decay rate, which is the same as that computed in part (b).

2. Consider the decay of the 3S state of Hydrogen to the 2P state and then to the 1S state.

   (a) To begin, ignore the electron spin. Then the 3S state is rotationally invariant, so that the angular distribution of the first emitted photon must be isotropic. Show this explicitly.

   (b) Even if the first photon is emitted isotropically, there can still be a correlation between the directions of the first and second photon. A strategy for computing this is to consider decays of the 3S to a photon along the $\hat{3}$ axis and then to compute the angular distribution of the second photon. Perform this calculation and find the distribution of the angle between the two photons.

(c) Now put back the electron spin. Show that the distribution of the first photon from a particular 3S state (for example, $j^3 = +1/2$) is still isotropic.

(d) Compute the distribution of the angle between the two photons for the two cases of decay through the $2P_{1/2}$ and $2P_{3/2}$ states.