# Physics 134 – Problem Set # 5

(due Thursday, May 17)

1. Consider a two-level system subject to an off-diagonal perturbation that has the form of a short but stronge pulse.

   (a) Let the wavefunction be represented by

   $$\Psi(t) = \begin{pmatrix} \alpha(t) \\ \beta(t) \end{pmatrix}$$

   and the unperturbed Hamiltonian be

   $$H_0 = \begin{pmatrix} -E_0/2 & \\ & E_0/2 \end{pmatrix}$$

   Find the general solution of the Schrödinger equation, and relate it to the problem of a spin in a magnetic field. If the initial condition is

   $$\Psi = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

   find $\Psi(t)$. What is the physical interpretation of this solution in the spin problem?

   (b) Add to the Hamiltonia a perturbation

   $$V = \begin{pmatrix} 0 & \mathcal{V}/\Delta \\ \mathcal{V}/\Delta & 0 \end{pmatrix}$$

   that is turned on at $t = 0$ and turned off at $t = \Delta$. Consider the limit $E_0\Delta \ll 1$. Show that, in this limit, the effect of the perturbation is independent of $\Delta$. This defines a perturbation that is a delta function in time. Given the initial condition

   $$\Psi = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

   compute the time-dependent wavefunction $\Psi(t)$ that results from this perturbation

   (c) Design a delta function perturbation, to be applied after a time $T$, that reverses the perturbation in (b) and restores the initial state (maybe up to a phase). Consider in particular the cases $T = 2\pi/E_0$, $T = \pi/E_0$, and $T = \pi/2E_0$.

2. In class, we studied the Hamiltonian

$$H = \begin{pmatrix} E_a & V\cos\omega t \\ V\cos\omega t & E_b \end{pmatrix}$$

using time-dependent perturbation theory. We found that the dominant terms were those in which the transition is driven near the resonant frequency. This motivates making the approximation

$$H \approx H_r = \begin{pmatrix} E_a & \frac{1}{2}Ve^{i\omega t} \\ \frac{1}{2}Ve^{-i\omega t} & E_b \end{pmatrix}$$

Griffiths (problem 9.7) discusses this approximation, called the *rotating wave approximation*

(a) Show that $H_r$ is Hermitian. If the wavefunction is given as in problem 1(a), what does this imply for $(|\alpha|^2 + |\beta|^2)$ ?

(b) Find the exact solution to the Schrödinger equation for $H_r$ with initial condition

$$\Psi = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

Your solution should involve the *Rabi flopping frequency*

$$\omega_r = \frac{1}{2}[(E_b - E_a - \omega)^2 + V^2]^{1/2}.$$

(c) For $\omega = (E_b - E_a)$, find the time at which the system is entirely in the state $|b\rangle$. What happens after that?

(d) For $\omega \neq (E_b - E_a)$, find the maximum of the probability that the system is in $|b\rangle$.

(e) Compute the probability that the system is in $|b\rangle$ as a function of $t$ and $\omega$ to first order in time-dependent perturbation theory. Compare to the exact solution. What is the condition on $V$ that the perturbation is "small"?