Physics 134 – Problem Set # 4

(due Thursday, May 10)

1. Here are some problems in the use of the Laplace transform:

   (a) Solve the free particle Schrödinger equation in 3 dimensions
   $$i\frac{\partial}{\partial t}\psi = -\frac{1}{2m}\nabla^2\psi$$
   with the initial condition
   $$\psi(\vec{r}) = \delta(\vec{r})$$
   using the Laplace transform. It is easiest to also Fourier transform in $\vec{r}$ and then transform back at the end of the analysis.

   (b) Let τ be a small fixed time. Solve the delay-differential equation
   $$\frac{1}{\tau^2}(x(t+2\tau) - 2x(t+\tau) + x(t)) = -\Omega^2 x(t)$$
   using the initial condition
   $$x(t) = X \qquad 0 < t < 2\tau$$
   It is sufficient to use the approximation $\Omega\tau \ll 1$. Show that the solution is unstable.

2. Solve for the scattering amplitude in two generalizations of the Wigner-Weisskopf problem presented in class. For $\lambda \ll 1$, find the partial wave expression for the scattering amplitude.

   (a) Let the added states be a multiplet of $L = 1$ states $|a\rangle$, with $a = x, y, z$. Write the transition matrix element as
   $$\langle\vec{r}|\, H\, |a\rangle = \lambda r^a \chi(r)$$
   where $\chi(r)$ is a spherically symmetric function. Show that the correct factor $(2\ell + 1) = 3$ appears in the partial wave formula.

   (b) Let the electron have spin, so that the incoming electron states are labeled by a spin quantum number $m = -\frac{1}{2}$ or $+\frac{1}{2}$. Let the added states be a set of states with $L = 1$ and $S = \frac{1}{2}$, summing to $J = \frac{1}{2}$. These states can be labeled $|M\rangle$, with $M = -\frac{1}{2}, \frac{1}{2}$. A rotationally invariant form for the transition matrix element is
   $$\langle\vec{r}, m|\, H\, |M\rangle = \lambda(\sigma^a)_{mM} r^a \chi(r)$$
   where $\chi(r)$ is rotationally symmetric, σ^a is a Pauli sigma matrix, and m and M are the two indices of the Pauli sigma matrix. The final answer will be more transparent if you use the Clebsch-Gordan coefficients for coupling spin $\frac{1}{2}$ plus spin 1 to spin $\frac{1}{2}$.