

Physics 134 – Problem Set # 3

(due Tuesday, April 24)

1. In the previous lecture, we computed the form factors associated with some simple charge distributions. The results were interesting: A smooth distribution of charge gave a monotonically decreasing form factor, while a distribution with a sharp edge gave an oscillating function with zeros that would be seen as zeros in the cross section. This change of form is relevant to electron scattering from nuclei and to mixed-phase materials with small bubbles inside a substrate. In both cases, the form factor is very sensitive to the thickness of the surface region. By evaluating integrals numerically, you can explore this phenomenon in more detail.

- (a) Let $\rho(r)$ be a spherically symmetric charge distribution such that

$$\int d^3x \rho(r) = 1$$

Write an expression for the form factor $F(Q)$ as a 1-dimensional integral over r .

- (b) An example of a normalized $\rho(r)$ is

$$\rho(r) = \frac{1}{(2\pi a^2)^{3/2}} \exp[-r^2/2a^2]$$

Evaluate the form factor analytically, and plot the result for $a = 1$.

- (c) Using your favorite mathematical software (Mathematica, MatLab, etc.), evaluate the form factor in (b) for $a = 1$ a number of values of Q by evaluating the integral in (a) numerically. You will have to cut off the integral over r at some large value. Show that it makes a negligible difference to integrate from 0 to 4 rather than from 0 to ∞ . Can we make the upper limit even smaller? Perform the same check of the numerical against the analytical result for $a = 0.1$.
- (d) Now consider the charge distribution

$$\rho(r) = A \frac{1}{1 + e^{b(r-a)}}$$

This has a relatively narrow surface region at $r = a$; the size of this region is set by b . The coefficient A should be adjusted so that the charge distribution is normalized. Fix $a = 1$, and begin with a small value of b , $b = 0.2$. Evaluate the form factor numerically. Does it have a zero as a function of Q ?

- (e) Evaluate the form factor for some larger values of b , up to $b = 5$, and understand the evolution of the form factor as a function of b .

2. Consider scattering from a spherical well potential

$$V(r) = \begin{cases} -A & r < a \\ 0 & r > a \end{cases}$$

where $A > 0$.

- (a) Write the equation for the function $g_{k0}(r) = rf_{k0}(r)$, where $f_{k0}(r)$ is the radial wavefunction. Justify the boundary conditions: $g_{k0}(r) = 0$ at $r = 0$, and $g_{k0}(r)$ and its first derivative are continuous at $r = a$.
- (b) Solve for the bound states in the $\ell = 0$ partial wave in this potential. Show that there are no bound states when A is very small, and that there are arbitrarily many bound states as A is made sufficiently large.
- (c) Compute the phase shift $\delta_0(k)$ as a function of A .
- (d) Fixing $\delta_0(k)$ so that $\delta_0(k) \rightarrow 0$ as $k \rightarrow \infty$, find the limit of $\delta_0(k)$ as $k \rightarrow 0$. Show that this limit flips from $\delta_0(0) = 0$ to $\delta_0(0) = \pi$ to $\delta_0(0) = 2\pi$, etc., as A is increased from 0. How are the values of A found here related to the results of part (b)?