

Physics 134 – Problem Set # 1

(due Tuesday, April 10)

1. Consider the problem of a particle of charge q_1 scattering from a very heavy particle of charge q_2 through electrostatic attraction or repulsion. This is called *Rutherford scattering*. We will study the quantum theory of this problem later in the course, but you can already work out the classical cross section, maybe by referring to your classical mechanics notes from last year. (This is problem 11.1 in Griffiths.) Assume that the heavy particle does not recoil, so the problem described by a static potential

$$V(r) = \frac{q_1 q_2}{4\pi\epsilon_0 r} .$$

- (a) In the repulsive case, the path of the particle is a hyperbola that bends away from the center of force. Construct this hyperbola, and show that the relation between the scattering angle and the impact parameter is

$$b = \frac{q_1 q_2}{8\pi\epsilon_0 E} \cot \frac{\theta}{2}$$

- (b) In the attractive case, the path of the particle is a hyperbola that bends toward the center of force. Show that the same relation between b and θ holds, with $q_1 q_2$ replaced by $|q_1 q_2|$.
- (c) Derive the differential cross section, and show that

$$\frac{d\sigma}{d\cos\theta} = \frac{q_1^2 q_2^2}{128\pi\epsilon_0^2 E^2 \sin^4 \frac{\theta}{2}} .$$

- (d) Show that the total cross section for Rutherford scattering is infinite. Is this infinity something we should worry about?

2. Picture a molecule as a rod of length L (measured in Å), and think about scattering high-energy electrons from this molecule. As an idealization, assume that all high-energy electrons that come within a distance r of the axis of the rod knock out some atomic electrons, thus resulting in an inelastic collision. Using this picture, we can compute the total cross section of the molecule.

- (a) First, imagine that the molecule is held in a fixed orientation, at an angle φ with respect to the electron direction. Compute the cross section as a function of φ .

- (b) Of course, no molecule can stay in a fixed orientation. This violates the Heisenberg uncertainty principle. If the molecule is in a gas, the simplest assumption is that it has a random orientation. Compute the total cross section for this case. You may assume that the orientation of the molecule is fixed during the collision. That is, assume that the high-energy electrons come in and do their damage before the molecule has a chance to rotate to a different orientation.
- (c) More properly, the molecule will be in some eigenstate of angular momentum, described by a wavefunction

$$\psi(\vec{n}) = Y_{\ell m}(\vec{n})$$

where \vec{n} is a unit vector pointing along the rod. Choose the quantization axis to be the direction of the incoming high-energy electrons. (If the molecule has some charge separation, the rotating molecule will have a magnetic moment, and we could use a magnetic field to favor some of these eigenstates over others.) Using the same assumption that the electron interacts with the molecule before it can change its orientation, compute the total cross section for all states with $\ell = 0, 1, 2$ and fixed m .

- (d) Average the results of part (c) over m for fixed ℓ . Explain the results. Does this regularity hold for all ℓ ?