Physics 134 - Midterm Exam

1. (40 points) Consider a scattering process involving a high energy particle hitting a target nucleus. The scattering involves a $\ell = 0$ component with phases shift smoothly dependent on energy

$$\delta_0(k) = -ka$$

such that $\delta \ll \pi/2$ over the whole energy range considered, an $\ell = 1$ component with a resonance (where the phase shift goes from 0 to πover a region of a few MeV) centered at $E = 10$ MeV, and an $\ell = 2$ component with a resonance centered at 20 MeV.

   (a) Sketch the total cross section as a function of energy from 0 to 30 MeV.

   (b) Sketch $d\sigma/d\cos\theta$ as a function of energy from 0 to 30 MeV at $\theta = 0$.

   (c) Sketch $d\sigma/d\cos\theta$ as a function of energy from 0 to 30 MeV at $\theta = 90°$.

   (d) Sketch $d\sigma/d\cos\theta$ as a function of $\cos\theta$ at an energy just below 20 MeV.

2. (60 points) Consider the scattering of quantum particles in 1 dimension from a series of repulsive delta function potentials $V(x) = V\delta(x - x_i)$. Consider scattering an energy $E = k^2/2m$. The answer will involve the combination

$$W = 2mV/k$$

   (a) Consider the case of one delta function potential at $x_1 = 0$. Solve the Schrödinger equation for the situation of a wave

$$\varphi(x) = \begin{cases} e^{ikx} & x < 0 \\ Ae^{ikx} + Be^{-ikx} & x > 0 \end{cases}$$

   Solve for A and B.

   (b) A and B satisfy a unitarity relation. What is it? Why?

   (c) Solve the Schrödinger equation for $x_1 = 0$ and the situation of a wave

$$\varphi(x) = \begin{cases} e^{-ikx} & x < 0 \\ Ce^{ikx} + De^{-ikx} & x > 0 \end{cases}$$

   Solve for C and D.

   (d) Write out the matrix

$$T = \begin{pmatrix} A & C \\ B & D \end{pmatrix}$$

   and compute the inverse of this matrix. How is this related to the reflection and transmission coefficients for this potential?

(e) Next consider the case in which the potential consists of two delta functions, at $x_1 = 0$ and at $x_2 = a$. Find the transmission and reflection probabilities for a particle of momentum k scattering from this potential.

(f) Find the condition on k such that there is zero probability of reflection.

At your leisure, you might want to think about the solution to this problem for a potential consisting of N equally spaced delta functions, in particular, in the limit where N is large.