Physics 211

Quantum Mechanics - Part 3

Michael E. Peskin

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OUTLINE:

- Basic Notions of Scattering Theory
- Scattering from a Central Potential
- Unitarity and the Optical Theorem
- Theory of an Unstable Quantum State
- Fermi's Golden Rule
- Interaction of Atoms with Light
- Thomson Cross Section
- Beta Decay
- Pion-Pion Scattering
- Energy Bands in Solids

1 Introduction

Quantum Mechanics is a difficult and counterintuitive subject. From your first introduction to physics, you are taught that particles can be approximated as points, moving along trajectories in accord with Newton's Laws. In Quantum Mechanics you are told that, no, that is not correct at all, that both the equations and the basic dynamical objects are completely different from anything you might have expected.

By now, I hope, you have gotten used to this. You have learned

- that particles in Quantum Mechanics are also waves, and vice versa, that neither of these descriptions is actually complete, and in what circumstances each description is the better approximation.
- that observables with definite values are replaced in Quantum Mechanics by operators, changing, philosophically, the nature of measurement.
- that Quantum Mechanics leads to discrete spectra of bound states, and how to compute the definite energies that appear in these spectra.
- that spin is quantized, with a variety of surprising consequences, including the stability of atoms.

This might seem to be enough to swallow in one year.

Still, something is missing. Almost all of the quantum mechanical results that you have studied up to this point in the course are static. You have diagonalized Hamiltonians to find spectra of discrete states, but nothing has been said so far about the transitions between these states, the creation of one of these states from parents, or the decay of unstable states into more stable products. Those topics will be the subject of this third quarter of the Quantum Mechanics sequence.

The basic paradigm fordescribing all time-dependent phenomena in Quantum Mechanics is scattering. This is an unfamiliar concept at first, but we have to get used to it. In classical mechanics, we derive time-dependent predictions by following individual particle trajectories. In quantum mechanics, even if there were no difficulties of principle here, in practice, it would be impossible. Atoms are *very small.* It is just barely possible now, in very special systems, to probe an individual atom to learn its properties, or to address an individual atom with external electric and magnetic fields. More often, we are forced to use a different technique. We throw a bunch of particles at an atom, and we see what particles come out. The initial particles may be other atoms, electrons, or photons. All of the information we have about the struck atom is contained in the distribution of particles that come away from the interaction. For example, the way that we measure the spectrum of bound states of an atom is to shoot it with a beam of light – a collimated bunch of photons – and then to measure at what wavelengths these photons are absorbed or scattered.

In this course, we will study the process of scattering formally and through explicit examples. We will try to undertstand how the microscopic processes involved in scattering lead to the final results, and how to decode those results in terms of underlying microscopic dynamics.

The plan of the course is the following: In the first few weeks, we will make a detailed study of the scattering of quantum mechanical particles from a fixed potential. We will related the process of scattering to Schrödinger wavefunctions, so that we can compute the results of a scattering process using formalism already familiar to you.

Armed with this knowledge, we will investigate in detail the interaction of an unstable quantum state with its decay channels. That discussion, I hope, will bring together, arguments from exact diagonalization of the Hamiltonian and arguments for various forms of perturubation theory. It will also illuminate the mysterious Heisenberg uncertainty relation between energy and time. The final result of this analysis will be a very useful approximation method, Fermi's Golden Rule.

In the final part of the course, we will apply Fermi's Golden Rule and other methods to a variety of problems in atomic, nuclear, and particle physics involving time-dependent interactions and unstable particles.

At the end of the course, I will describe the special situation of quantum mechanics in a periodic potential and introduce the band structure of electrons in solids.

The material for this course will be drawn from many textbooks and other sources. Useful references are the textbooks:

- D. Griffiths, Introduction to Quantum Mechanics
- S. Gasiorowicz, Quantum Physics
- C. Cohen-Tannoudji, B. Diu, and F. Laloë, Quantum Mechanics
- K. Gottfried and T. M. Yan, Quantum Mechanics: Fundamentals
- G. Baym, Lectures on Quantum Mechanics
- L. Schiff, Quantum Mechanics

More information about these books can be found on the course web page

`http://www.slac.stanford.edu/~mpeskin/Physics134/`

In these notes, I will try to keep a consistent set of notational conventions. Here is a brief introduction:

I will denote a 3-vector as $\vec{V}$. I will denoted the unit vector in the direction of this vector as $\hat{V}$. At some points, we will need to use indices for vectors:

$$\vec{V} \longrightarrow V^k \qquad k = 1, 2, 3$$

You should be familiar with summation convention, that repeated indices are summed over $(1, 2, 3)$ or whatever the natural values are. This leads to identities such as

$$\epsilon^{ijk}\epsilon^{mnk} = \delta^{im}\delta^{jn} - \delta^{in}\delta^{jm}$$

I will distinguish operators from c-numbers, where such clarification is necessary, by an under-tilde:

$$\underset{\sim}{H} \qquad \underset{\sim}{\vec{P}} \qquad \underset{\sim}{\vec{J}}$$

I will use the convention for Fourier transforms:

$$f(x) = \int \frac{dk}{2\pi} e^{ikx} \tilde{f}(k) \qquad \tilde{f}(k) = \int dx\, e^{-ikx} f(x)$$

Note that each integral over the Fourier transform variable comes with a factor $1/(2\pi)$. This is very useful to avoid dropping factors of 2π. The delta functions for coordinates and Fourier transform variables should appear in the form

$$\delta(x - x') \text{ or } \delta^{(3)}(\vec{x} - \vec{x}') \qquad 2\pi\delta(k - k') \text{ or } (2\pi)^3\delta^{(3)}(\vec{k} - \vec{k}')$$

I will consistently omit factors of $\hbar$. That is, I will work in units where $\hbar = 1$, so that momentum is measured in units of inverse length cm^{-1} and energy is measured

in units sec^{-1}. This will simplify all of our formulae. At the end of the day, you can restore the correct units by multiplying by the appropriate number of factors of

$$\hbar = 1{,}054572 \times 10^{-34}\ \text{J sec} \qquad \text{J sec} = \text{kgm}^2/\text{sec} \cdot \text{m}$$

We will not treat any relativistic topics in this course, so I will keep factors of c explicit. In deference to Mr. Griffiths, I will notate electrodynamics in SI units. That is, I will keep the (actually, meaningless) factors of ϵ_0 and μ_0 that appear in SI versions of electrodynamics formulae. Since

$$c = \frac{1}{(\epsilon_0 \mu_0)^{\frac{1}{2}}}$$

I will typically write formulae in terms of ϵ_0 and c.