Physics 134 - Final Exam

In the attached table of Clebsch-Gordan coefficients, the notation is:

- Each set of boxes gives the Clebsch-Gordan coefficients for $J_1 \times J_2$.
- The values of J, M are given at the top. The values of m_1, m_2 are given on the left side.
- The value of the Clebsch is the square root of the value given, with the minus sign then applied.

Then, for example, for $(J_1 = 2) \times (J_2 = \frac{1}{2})$,

$$\langle m_1 = 0, m_2 = 1/2 | J = 5/2, M = 1/2 \rangle = +\sqrt{3/5}$$

$$\langle m_1 = 0, m_2 = 1/2 | J = 3/2, M = 1/2 \rangle = -\sqrt{2/5}$$

1. (20 points) Consider a scattering process involving a spinless particle scattering from a nucleus. There are two sets of narrow resonances at the same energy of 20 MeV. One set has $\ell = 1$ and width $\Gamma = 8$ MeV; the other has $\ell = 2$ and width $\Gamma = 2$ MeV. You can assume that both of the relevant phase shifts are close to zero below the resonance, and you can ignore the contribution to the scattering from other partial waves.
   (a) What is the total cross section at $E = 20$ MeV? You can leave the result in terms of the particle momentum k at this energy.
   (b) Sketch the total cross section as a function of energy near the energy of 20 MeV.
   (c) Make careful sketches of the differential cross section $d\sigma/d\cos\theta$ as a function of $\cos\theta$ at the energies of 15, 19, 20, 21, and 25 MeV.

2. (20 points) Consider a spin-$\frac{1}{2}$ particle of charge q, mass m, and g-factor g in a magnetic field $\vec{B} = B_0\hat{3}$. The spin starts in its ground state at $t = 0$. Then a small B field in the $\hat{1}$ direction is ramped up smoothly with time according to

$$B^1(t) = bt$$

   (a) What are the energies of the ground state and the excited states of the spin before the B^1 field is turned on?
   (b) Using first order time-dependent perturbation theory compute the probability that, after a time T, the spin is in the excited state.
   (c) A naive expectation might be that the amplitude for a transition increases for large t like $\int dt\, t \sim t^2$, giving a probability of a transition growing like t^4. What is the correct result? Why is it different?

(d) At what time t does the approximation of using first-order time-dependent perturbation theory break down?

3. (30 points) A photon beam interacts with an electron in an atomic state with $L = 2$, $J = \frac{3}{2}$ by an E1 transition. Several resonances are seen, and we would like to work out the L and J of each.

   (a) Write out the $L = 2$, $J = \frac{3}{2}$, $J^3 = \frac{3}{2}$ and $J^3 = \frac{1}{2}$ states in terms of states with definite m, s^3.

   (b) What are the possible values of L for the resonances? What combinations of (L, J) are possible consistent with the selection rules?

   (c) Now imagine that we could polarize the photon and also polarize the initial atomic state. Consider, in particular, the initial state in which the photon has $\vec{\epsilon} = \vec{\epsilon}_+$ $(S^3 = +1)$ and the electron in the atom has $J^3 = +3/2$. Among the possibilities for the resonances, which are seen in this polarized reaction, and which are not?

   (d) For the resonance associated with the transition to the $J = \frac{5}{2}$ state, work out the angular distribution for the scattering of unpolarized photons from unpolarized atoms.

4. (30 points) In nature, there is a heavy lepton τ^- that decays to $\pi^-\nu$ and $\rho^-\nu$, among other channels. In this problem, you will study a simplified model of these decays. Consider, then, a heavy elementary particle T that decays to another heavy elementary particle P and a neutrino ν, or to a heavy elementary particle R and a neutrino

$$T \to P + \nu \qquad T \to R + \nu$$

   Assume that the mass difference between P and T and between R and T is much less than any of the masses, so that you can ignore recoil. Let T be a state of $J = \frac{1}{2}$. Let P be a state of $J = 0$ and let R be a state of $J = 1$. Let the neutrino be massless and have spin $J = \frac{1}{2}$. In the strong and electromagnetic interactions, T, P, and R are found to have parity $+1$, -1, -1, respectively.

   (a) A possible decay matrix element for $T \to P\nu$ is

$$\left\langle P\nu(\vec{p}, s^3)\right| \Delta H \left|T(s^3_T)\right\rangle = [A\mathbf{1} + B\hat{p}\cdot\vec{\sigma}]_{s^3\, s^3_T}$$

   where the quantity in brackets is a 2×2 matrix, with $\mathbf{1}$ the identity matrix and $\vec{\sigma}$ the Pauli sigma matrices. For the case $A = 0, B \neq 0$, find the parity of the ν. Similarly, for the case $B = 0, A \neq 0$, find the parity of the neutrino.

   (b) Compute the decay width for the decay $T \to P\nu$ in terms of A and B and the energy difference $(m_T c^2 - m_P c^2)$.

   (c) In nature, it turns out (for the more realistic reaction $\tau \to \pi\nu$) that $B = -A$. Odd, but true. For this case, compute the angular distribution of the neutrino for a T polarized such that $s^3_T = +\frac{1}{2}$. This angular distribution can be measured in the real system by observing the π^- recoil.

(d) Why does this angular distribution imply that parity is violated?

(e) A possible decay matrix element for $T \to R\nu$ is

$$\left\langle R(\vec{\epsilon})\nu(\vec{p}, s^3)\right| \Delta H \left|T(s_T^3)\right\rangle = C[(\mathbf{1} - \hat{p} \cdot \vec{\sigma})(\vec{\epsilon} \cdot \vec{\sigma})]_{s^3\, s_T^3}$$

Compute the decay width for the decay $T \to R\nu$ in terms of C and the energy difference $(m_T c^2 - m_R c^2)$. Note that it is necessary to sum over the possible polarization states of R in the final state.

(f) For the decay $T \to R\nu$, compute the angular distribution of the neutrino for a T polarized so that $s_T^3 = +\frac{1}{2}$.