

Sample Exam Question from Physics 134 - Advanced Topics in Quantum  
Mechanics

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Question:

A heavy particle such as the top quark decays by weak interactions to a tau lepton, ejecting this particle at close to the speed of light. The tau then decays to a pi meson and a neutrino. In that decay, the pi meson goes off with a fraction  $f$  of the energy of the tau, and the neutrino gets the rest. What is the probability distribution of  $f$ ?

Answer:

$(1 - f)$ . (“one minus f”). This is zero when  $f = 1$ . The pi meson is not permitted to take away the full energy of the tau.

The correct answer would be followed by a shrug of the shoulders and the statement: “It is a simple consequence of the conservation of angular momentum.”

(For sticklers, the full correct answer is  $2(1 - f)df$ , ignoring the mass of the pion. The above is quite sufficient.)

Here are some further notes that give background on this question. I am sure that the ESPN Game Day announcers do not want to know all of this, but maybe some tidbits will be interesting to them.

1. **What is a tau lepton?** The tau ( $\tau$ ) lepton is an elementary particle identical to the electron in all respects except that it is 3500 times heavier. No one knows why there is such a particle. The tau, and the decay process described in this problem, were discovered in 1974 at Stanford’s giant particle accelerator SLAC. Professor Martin Perl made the discovery and won the Nobel Prize for this achievement.
2. **Why is it forbidden for the pi meson to take all of the energy of the tau?** It is not possible to throw an elementary particle without putting spin on it. Physicists describe particles as right-handed or left-handed according to their direction of spin. Point your thumb in the direction that the particle is moving; then the fingers of the right or left hand curl to indicate the direction of right- or left-handed spin. (A right-handed quarterback puts spin on the football in the direction that physicists call left-handed.) It is a fact of nature that particles created by the weak interaction always spin in the left-handed direction. Also, neutrinos, which travel at almost the speed of light, are always left-handed. So when a left-handed spinning tau decays to a left-handed spinning neutrino, the neutrino goes forward, to preserve the spin angular momentum. The neutrino typically carries off a large fraction of the tau’s original energy, and it always carries off some nonzero amount.

In quantum mechanics, the amount of spin of a tau or a neutrino is a definite number, equal to Planck's constant divided by  $4\pi$ . Each definite value of the spin leads to its own decay pattern, described by simple functions called "spherical harmonics". Much of our Quantum Mechanics course is devoted to understanding these functions and their applications.

3. **Why is the answer given as a probability?** In quantum mechanics, it is not possible to predict a definite outcome for a specific particle decay. For example, for any given tau decay, it is not possible to predict the definite value of  $f$ . We only know that, if we watch many taus decay, the distribution of outcomes will follow the law that is given in the answer to the problem. The fact that answers to questions in quantum mechanics are probabilities is part of the content of the "Heisenberg uncertainty principle". Most people are familiar with the random nature of radioactive decay processes. This is why Geiger counters stutter rather than giving a uniform, steady stream of clicks, and why a radioactive decay rate is quoted as a 'half-life'.
4. **Why would anyone want to know the answer to this exam question?** Physicists today are trying to use spin and angular momentum formulas like the one in the exam problem to find out what kind of particle makes up the mysterious dark matter that composes 80% of the matter in the universe. This is not a 'practical' application, but it is an interesting one. The same formulas find more down-to-earth application in explaining why semiconductors have the special properties used in computer chips and why lasers work.
5. **Why did you give us so much information?** Why do you think? I'm a physics professor.

Best wishes,

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