

# Physics 130

## Quantum Mechanics - Part 1

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Stanford University, Winter Term 2013

### OUTLINE:

- Schrödinger Equation and Energy Quantization
- Foundations of Quantum Mechanics
- Simple Quantum Systems; Spin
- Entanglement; Interpretation of Quantum Mechanics

## 1 Introduction

Physics 130 is the beginning of your introduction to quantum mechanics, the framework for the description of physics in the small. Quantum Mechanics applies from distances of the sizes of molecules ( $10^{-9}$  cm) to the smallest distances yet studied experimentally ( $10^{-18}$  cm). More correctly, quantum mechanics is the correct framework for expressing the laws of physics at all scales from the very largest to the very smallest, though the predictions of quantum mechanics are close to those of classical mechanics for macroscopic objects.

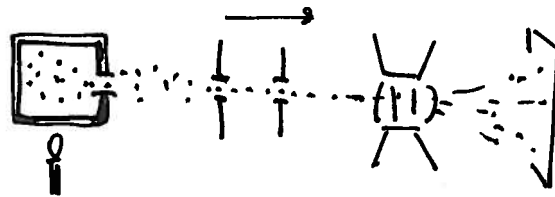
Classical mechanics is taught in freshman physics because it is based on concepts that are easily visualized: point particles interacting through forces. Even in classical electrodynamics, which is based on fields that permeate all of space, it is not so difficult to visualize patterns of arrows that give the electric and magnetic fields through which particles move. Quantum mechanics is completely different. The basic objects of study in quantum mechanics are the quantum states of particles, whose properties are unexpected and counterintuitive. You are familiar hearing that quantum states have the properties of both particles and waves, yet neither description applies consistently. The main purpose of this course is to explain what a quantum state is, and it will take me essentially the entire term to do so.

There are not many courses at Stanford — outside of religious studies and, maybe, the Hoover Institute — where the professor says at the beginning that everything you have learned previously in this subject is a lie and now he will tell you the truth. But, that is exactly the outline of the first course in quantum mechanics. You will need this knowledge. Only with an understanding of quantum states can you go on to understand the detailed workings of atoms, nuclei, and elementary particles and forces.

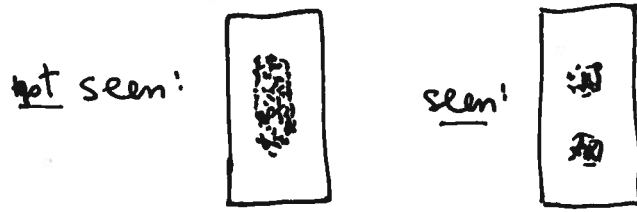
The mathematics in this course will be rather heavy. I am sorry, but this is the price of reality. Quantum mechanics is based on mathematical ideas that had not been discovered in the time of Galileo and Newton — linear algebra, infinite-dimensional vector spaces, transformation groups. I will try to give as gentle an introduction to these ideas as possible.

We would not throw away so successful a theory as classical mechanics without a good reason. So, I would like to begin this course by listing ten phenomena that would be completely impossible in a world based on classical physics. Put yourself, then, in the nineteenth century world of point particles and classical electromagnetic fields. Here are ten things that could not happen in that world, but do happen in ours.

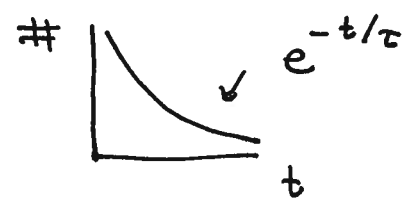
1. **The Stern-Gerlach experiment:** This is an experiment first performed in 1922 by Otto Stern and Walter Gerlach that gives a characteristic example of the weirdness of quantum phenomena. Heat up silver in an oven, and extract a beam of silver atoms. Silver (atomic number 47) has one unpaired electron. As a result of this, the atom has a measurable magnetic moment. Classically, this moment would be a vector that points in a random orientation. Now pass the beam through a magnetic field that bends upward atoms with an upward-pointing magnetic moment and bends downward atoms with a downward-pointing magnetic moment. Capture the atoms on a piece of paper:



Classically, the expectation would be that the silver atoms would make a single extended spot reflecting the variety of orientations. Instead, what is seen is two spots corresponding to two (and only two) distinct orientations.



2. **The discreteness of light:** At macroscopic brightness, light seems continuous in strength. If we have a source of light and a detector, the response of the detector is proportional to the intensity of the illumination. However, at very low levels of illumination, something different is seen. The *response* of the detector is fixed (for light of a fixed frequency or color), but the *rate of responses* decreases. This suggests that light has some properties of a particle, carrying energy in fixed amounts.
3. **Radioactive decay:** Certain nuclei are unstable and decay by emitting high-energy particles, alpha or beta radiation. For a macroscopic sample, the rate of emission of radiation follows an exponential decay law



However, the actual time at which any individual nucleus decays cannot be predicted. These times are apparently random, subject only to a law of equal probability per unit time. You are familiar with this: A Geiger counter does not click at a uniform rate but rather with clicks spaced randomly in time.

4. **Discreteness of atomic spectra:** A gas of atoms, heated in a flame to thousands of degrees C, emits light. This light is not emitted at all frequencies but rather at a set of specific frequencies that depends on the identity of the atom. For example, sodium vapor in a flame emits light at a characteristic yellow of wavelength 5893 Å. In vacuum, light can have any frequency, so this must be a property of the atom, that only specific energies are allowed. It is difficult to reconcile this with a classical picture of an atom as a nucleus surrounded by a cloud of orbiting electrons.
5. **Stability of atoms:** In fact, it is clear from classical mechanics that atoms should not exist at all. An electron orbiting a nucleus will give off radiation, lose energy, and spiral into the center. In astronomy, we see this process of radiation and capture for objects in the neighborhood of a black hole at the center of an active galaxy. But, atoms do not collapse in this way; they are stable for the lifetime of the universe.

6. **Conductivity of metals:** Some elements form solids that are *insulators* with very high resistivity. This corresponds to the property that free electrons in the solid cannot move very far without hitting some atom and being slowed down. Solid Chlorine ( $\text{Cl}_2$ ) or Argon has this property. However, the next elements in the periodic table, Potassium and Calcium, form shiny metals with excellent conductivity. Electrons in these solids can travel thousands of atomic spacings without scattering off of anything and being slowed down. In a classical picture in which electrons fill the spaces between nuclei, how is this possible?
7. **Chemical bonds:** Two atoms of Hydrogen can lower their energy by exchanging electrons to form a Hydrogen molecule  $\text{H}_2$ . Similarly, many other pairs of atoms easily form chemical bonds. Classical mechanics might give some idea of why the energy is lowered, but it cannot answer other questions: Why do bonds between two atoms always give the same lowering of energy. Why do bonds have definite orientations? Why are bonds formed between pairs of atoms, rather than having the group of atoms in the molecule interact promiscuously?
8. **Spin of the electron:** As we noted above in discussing the silver atom, an isolated electron has a small but detectable magnetic moment. The size of this moment is

$$\mu = e \cdot (2 \times 10^{-11} \text{ cm}) \cdot c$$

Such a magnetic moment might be formed if the electron were a spinning sphere of charge of size  $10^{-11}$  cm, with the edge of the sphere spinning at the speed of light. But we know that the electron is not that large. Electrons interact with atomic nuclei (of size  $10^{-13}$  cm) as point particles. Where, then, can the angular momentum come from? What makes this more puzzling is that all electrons are seen to have the same magnetic moment, to an accuracy of 10 decimal places!

9. **Identity of electrons:** Electrons are identical not only in their magnetic moments but in all other properties – mass, charge, etc. We might explain this by the fact that electrons are elementary. However, other particles that we know to be composite – the  $\text{He}^4$  nucleus, the proton – also have masses that are the same to very high accuracy between different representatives. Also, all of these particles have antiparticles with the same mass and the opposite charge.
10. **Superconductivity:** The problem of the conductivity of metals becomes much harder to understand from the phenomenon that many metals (e.g., Mercury, Lead, Aluminum) have a phase transition at a few degrees K below which their resistivity is *zero*. Literally, electrons can go thousands of miles through these

materials without being scattered. A related phenomenon is that liquid He<sup>4</sup> flows frictionlessly below 2.2° K. We cannot begin to describe these phenomena in classical mechanics. Even in quantum theory, they require quantum states of macroscopic extent.

None of these phenomena can be accounted for by the physics of the nineteenth century. Yet, with quantum mechanics, all have ready explanations, most of which we will discuss in this course. So, it is time to begin.

The material for this course will be drawn from many textbooks and other sources. Useful references are the textbooks:

- D. Griffiths, Introduction to Quantum Mechanics, 2nd ed.
- R. Shankar, Principles of Quantum Mechanics
- J. Townsend, A Modern Approach to Quantum Mechanics
- R. Feynman, R. Leighton, and M. Sands, The Feynman Lectures on Physics, vol. 3
- S. Gasiorowicz, Quantum Physics
- C. Cohen-Tannoudji, B. Diu, and F. Laloë, Quantum Mechanics
- K. Gottfried and T. M. Yan, Quantum Mechanics: Fundamentals
- G. Baym, Lectures on Quantum Mechanics
- L. Schiff, Quantum Mechanics

More information about these books can be found on the course web page

<http://www.slac.stanford.edu/~mpeskin/Physics130/>

In these notes, I will try to keep a consistent set of notational conventions. Here is a brief introduction:

I will denote a 3-vector as  $\vec{V}$ . I will denote the unit vector in the direction of this vector as  $\hat{V}$ . At some points, we will need to use indices for vectors:

$$V^i \quad i = 1, 2, 3 \quad \text{or} \quad x, y, z$$

You should be familiar with summation convention, that repeated indices are summed over (1, 2, 3) or whatever the natural values are. This leads to identities such as

$$\delta^{ij} \delta^{jk} = \delta^{ik} \qquad \epsilon^{ijk} \epsilon^{ilm} = \delta^{jl} \delta^{km} - \delta^{jm} \delta^{kl}$$

I will distinguish operators from simple numbers ("c-numbers"), where such clarification is necessary, by an under-tilde:

$$\text{number : } x \qquad \text{operator : } \tilde{x}$$

I will use the convention for Fourier transforms:

$$f(x) = \int \frac{dk}{2\pi} e^{ikx} \tilde{f}(k) \qquad \tilde{f}(k) = \int dx e^{-ikx} f(x)$$

Note that each integral over the Fourier transform variable comes with a factor  $1/(2\pi)$ . This is very useful to avoid dropping factors of  $2\pi$ .

I will try to write factors of Planck's constant explicitly. Typically, we will use as our standard constant  $\hbar = h/2\pi$ . The natural set of units for quantum mechanics would set  $\hbar = 1$ , so that momentum is measured in units of inverse length  $\text{cm}^{-1}$  and energy is measured in units  $\text{sec}^{-1}$ . This will simplify all of our formulae. At the end of the day, you can restore the correct units by multiplying by the appropriate number of factors of

$$\hbar = 1.054572 \times 10^{-34} \text{ J sec} \qquad \text{J sec} = \text{kg m}^2/\text{sec}$$

I hope you will be ready for this by the end of the term.

We will not treat any relativistic topics in this course, so I will keep factors of  $c$  explicit. In deference to Mr. Griffiths, I will notate electrodynamics in SI units.

That is, I will keep the (actually, meaningless) factors of  $\epsilon_0$  and  $\mu_0$  that appear in SI versions of electrodynamics formulae. Since

$$c = \frac{1}{(\epsilon_0 \mu_0)^{1/2}}$$

I will typically write formulae in terms of  $\epsilon_0$  and  $c$ .