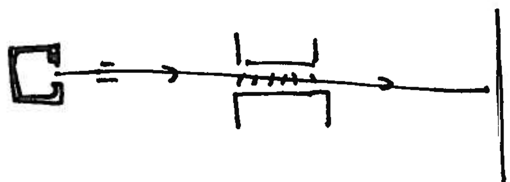
Quantum Coherence

Now we come to the most difficult and mysterious aspect of quantum mechanics, the effect of measurements. I will introduce this subject by describing the Stern-Gerlach experiment that I mentioned in the first lecture.

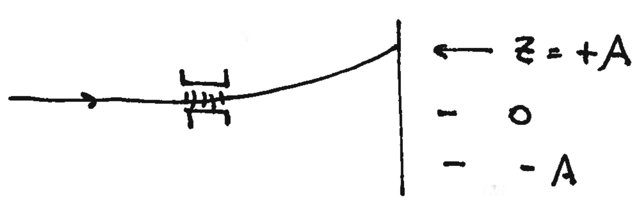
Consider an experimental setup in which we heat silver atoms in an oven, collimate them into an atomic beam, and direct them through an inhomogeneous magnetic field. Finally, the atoms move to a screen where they deposit and can be detected.



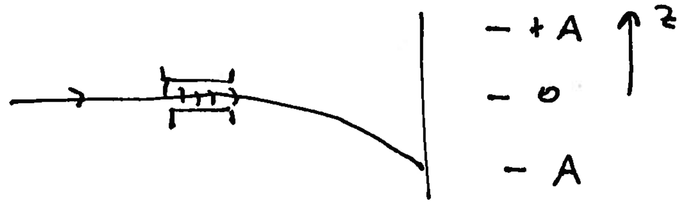
A silver atom has an even number of nucleons and an odd number of electrons. We can consider it to be a carrier for the final unpaired electron. The electron has a magnetic moment, so in an magnetic field the electron feels a potential

$$-\vec{\mu}\cdot\vec{B} = +\frac{e}{m}\vec{S}\cdot\vec{B}$$

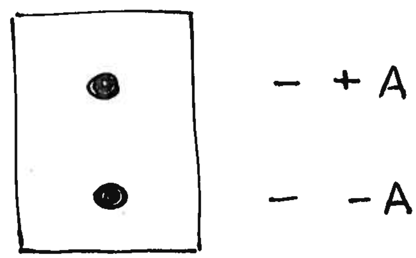
If the magnetic field varies from point to point, the electron feels a force. If $\vec{B} = B(z)\hat{z}$ and *decreases* with increasing z, then an electron with spin up ($S^z = +\hbar/2$) will be pushed up and an electron with spin down ($S^z = -\hbar/2$) will be pushed down. An atom carrying an electron with spin up will deposit on the screen at a position $z = +A$



An atom carrying an electron with spin down will deposit on the screen at position $z = -A$.



If a beam of atoms in which the initial electron spin states are randomly $|\uparrow\rangle$ and $|\downarrow\rangle$ passes through the apparatus, the pattern that appears on the screen will be



This result is already odd, but perhaps it is not so difficult to understand. The initial conditions in which the electron spin is aligned with the $\hat{z}$ axis is rather special. However, it is difficult to control the spin of an atom that comes out of the oven. More typically, the initial electron spin will be in a random direction. What then?

For definiteness, consider the case in which the initial electron spin is in the $\hat{x}$ direction. Then the spinor is

$$\frac{1}{\sqrt{2}}\begin{pmatrix}1\\1\end{pmatrix}$$

This is equal to

$$\frac{1}{\sqrt{2}}\left[\ |\uparrow\rangle + |\downarrow\rangle\ \right]$$

We already know that

$$|\uparrow\rangle \longrightarrow |\text{atom deposited at } z = +A\rangle$$

$$|\downarrow\rangle \longrightarrow |\text{atom deposited at } z = -A\rangle$$

The basic principles of quantum mechanics tell us that time evolution is a unitary, linear, transformation. Thus

$$\frac{1}{\sqrt{2}}\begin{pmatrix}1\\1\end{pmatrix} \rightarrow \frac{1}{\sqrt{2}}\Big[\,|\text{atom at } z = +A\rangle + |\text{atom at } z = -A\rangle\,\Big]$$

We can set up the experiment so that the points $z = +A$ and $z = -A$ are macroscopically separated. So this state is one with a linear combination of two macroscopically distinct outcomes. What does that mean?

In our experience, the silver atom can deposit at either $z = +A$ or $z = -A$, but, if there is only one atom, it cannot deposit itself at both places. What is actually observed to happen is that if the experiment is repeated with a large number of silver atoms, the atoms deposit at $z = +A$ with probability $\frac{1}{2}$ and at $z = -A$ with probability $\frac{1}{2}$.

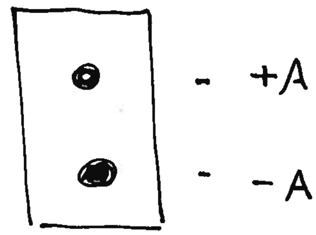
A general initial spin orientation can be written as

$$\xi = a\,|\uparrow\rangle + b\,|\downarrow\rangle \qquad |a|^2 + |b|^2 = 1$$

In this case, if the experiment is repeated many times, the atom deposits at $z = +A$ with probability $|a|^2$ and at $z = -A$ with probability $|b|^2$. In an ensemble of spins with random orientations,

$$\langle |a|^2 \rangle = \langle |b|^2 \rangle = \frac{1}{2}$$

Thus, a beam of randomly oriented silver atoms produces the pattern



That outcome is more than odd; it is definitely weird. However, this is actually what happens when one repeats the experiment (as I have personally observed).

In this lecture and the next, I will consider other examples in which we attempt to measure quantities $\mathcal{O}$ in states that are not eigenstates of $\mathcal{O}$. The outcomes follow this example. I would like to state the general rule clearly. Consider first the measurement of $\mathcal{O}$ in a state that is an eigenstate $|n\rangle$ of $\mathcal{O}$. The value that we obtain from this measurement will be the corresponding eigenvalue λ_n. To actually make the measurement, we carry out some procedure on the initial state that corresponds to time evolution with a well-chosen Hamiltonian. At the end of the procedure, the number λ_n appears on a dial or LED display. Schematically,

$$|n\rangle \longrightarrow |\text{measurement} = \lambda_n\rangle$$

Now start from a state $|\psi\rangle$ that is not an eigenstate of $\mathcal{O}$. To analyze the effect of the measurement, we can resolve $|\psi\rangle$ as a linear combination of $\mathcal{O}$ eigenstates

$$|\psi\rangle = \sum_n c_n |n\rangle$$

Now act with the measurement procedure. Each eigenstate evolves to a state in which its value is measured. Then

$$|\psi\rangle \longrightarrow \sum_n c_n \, |\text{measurement} = \lambda_n\rangle$$

This is the prediction of quantum mechanics. We obtain a state that is a linear superposition of states that have different macroscopic properties.

What we observe, though, is that each measurement can have only one outcome. The value measured is always one of the λ_n. If the experiment is repeated many times, the value λ_n appears with probability $|c_n|^2$, with

$$\sum_n |c_n|^2 = 1$$

The quantity c_n is called the *probability amplitude*; its square is the observed probability.

We describe this situation by saying that the state above *collapses* to one of the states $|n\rangle$. The state ψ collapses to the state $|n\rangle$ with probability $|c_n|^2$. Before collapse, we say that the various possible states are in a *coherent superposition.* After collapse, the various eigenstates are *incoherent.*

The idea of the collapse of a wavefunction raises many troubling issues: What is the mechanism of the collapse of a wavefunction? Is it a deterministic or intrinsically probabilistic process? Do wavefunctions "really" collapse? These are difficult questions, and professional physicists do not agree on the answers. I encourage you to go around to the faculty members in the Physics Department and ask them the following two questions:

1. Is the Schrödinger wavefunction real, or only a tool to compute probabilities?

2. Does the Schrödinger wavefunction actually collapse?

I predict that you will get the whole spectrum of answers to these questions. I will tell you my own opinion in the last lecture of the course. Until then, I would like to adopt the collapse of the wavefunction as provision description of measurements and explore its consequences.

The fact that quantum mechanics gives us a choice between strange wavefunctions that contain linear superpositions of states with different measurements and the strange phenomenon of collapse of the wavefunction was profoundly disturbing to two of the sharpest-thinking founders of quantum mechanics, Einstein and Schrödinger. I will discuss Einstein's objections in the last lecture of this course. Schrödinger was the first to point out clearly the troublesome nature of the wavefunctions that are predicted by quantum mechanics. I would like to quote at some length from his 1935 paper [Naturwissenschaften 23, 807 (1935), translated by J. D. Trimmer and reprinted in *Quantum Theory and Measurement*, J. A. Wheeler and W. H. Zurek, eds (Princeton, 1983)]:

> One can even set up quite ridiculous cases. A cat is penned up in a steel chamber, along with the following diabolical device (which must be secured against direct interference by the cat): in a Geiger counter there is a tiny bit of radioactive substance, *so* small that *perhaps* in the course of one hour one of the atoms decays, but also, with equal probability, perhaps none; if it happens, the counter tube discharges and though a relay releases a hammer which shatters a small flask of hydrocyanic acid. If one has left this entire system to itself for an hour, one would say that the cat still lives *if* meanwhile no atom has decayed. The first atomic decay would have poisoned it. The ψ-function of the entire system would express this by having the living and the dead cat (pardon the expression) mixed or smeared out in equal parts.
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> It is typical in these cases that an indeterminacy originally restricted to the atomic domain becomes transformed into macroscopic indeterminacy, which can then be resolved by direct observation. That prevents us from so naively accepting as valid a "blurred model" for representing reality. In itsef it would not embody anything uncleaer or contradictory. There is a difference between a shaky or out-of-focus photograph and a snapshot of clouds and fog banks.

The strict prediction of quantum mechanics in this example is that that quantum state of the box after one hour is:

$$\frac{1}{\sqrt{2}}\left[\; |\text{cat alive}\rangle + |\text{cat dead}\rangle \;\right]$$

Is this acceptable? Can you picture this state as actually existing? If the problem is resolved by collapse of the wavefunction, when does that collapse take place? When the cat dies, or when the human observer opens the box? These are serious questions, worth your careful introspection.