# Physics 124 – Problem Set # 6

(due Friday, November 8)

(All of these problems are cribbed from Heald and Marion.)

1. Consider Thomson scattering of unpolarized light propagating in the $\hat{z}$ direction. View unpolarized light as an equal mixture of light pulses linearly polarized along $\hat{x}$ and $\hat{y}$. Now consider the light scattered at an angle $\theta$ at $\phi = 0$, *i.e.*, the light scattered at an arbitrary angle in the $\hat{x}, \hat{z}$ plane. Show that the light scattered almost forward is unpolarized, but that the light scattered at $\theta = \pi/2$ is completely polarized. Define the polarization $p$ of the light scattered in an arbitrary direction as

$$p = \frac{P_\perp - P_\parallel}{P_\perp + P_\parallel} \,, \tag{1}$$

where $P_\perp$, $P_\parallel$ are the radiated powers with polarization vector perpendicular to and parallel to the $\hat{x}, \hat{z}$ plane. Compute $p$ as a function of $\theta$.

2. In Physics 120, we computed the electric dipole moment induced on a neutral perfectly conducting sphere placed in an electrostatic field. Use this result to compute the electromagnetic radiation scattered from such a sphere of radius $a$ in the limit $\lambda \gg a$. What is the cross section of the sphere as a function of $a$?

3. A charged particle in a harmonic oscillator loses energy by radiation. The rate of energy loss is proportional to the square of the amplitude; thus, the amplitude decreases by an amount linear in the amplitude. We can represent this damping as a term in the oscillator equation

$$\ddot{x} + \gamma_n \dot{x} + \omega_0^2 x = 0 \tag{2}$$

Compute $\gamma_n$ in terms of $e$, $\omega_0$, and $m$. This is the 'natural linewidth' of an atomic resonance. Compute $\gamma$ numerically for an electron in an atom with a transition energy $\Delta E = \hbar\omega_0 = 10$ eV.

4. The spectrum of mercury has a blue line at $\lambda = 435.8$ nm, a green line at 546.1 nm, and a yellow doublet at 577.0 and 579.1 nm. It is observed with a grating consisting of 40 slits. Discuss the appearance of the yellow doublet in the 3rd, 7th, and 17th orders (*i.e.*, at the maxima $m = 3, 7, 17$). Consider both resolution and overlapping of orders.

5. Consider a solid crystal, in which $N$ atoms or molecules are arranged in a simple cubic structure with separation $a$. Treat $N$ as a very large number (of order $10^{20}$). Assume that the light comes in parallel to one of the axes of the crystal.

(a) Assume first that the wave is scattered isotropically. This is a good assumption if the crystal is made of atoms and we consider scattering in directions close to the forward direction. To aid in thinking about this, let the wavelength of the incident light be much smaller than $a$ (*e.g.*, X-ray radiation with $\lambda = 1$ Å scattering from unusually large spherical atoms). Show that the radiation pattern is a set of discrete spots. Find the condition for the location in solid angle of the illuminated spots.

(b) Next, let each atom radiate as in Rayleigh scattering. Find the distribution of intensities of the spots.

(c) Finally, consider the case of a crystal of identical molecules with some complicated structure, for example, proteins. (Proteins really are large and justify the setup of part (a).) Find a formula for the relative intensities of the spots, in terms of the Fourier transform of the distribution of atoms in each molecule.