Physics 124 – Problem Set # 4

(due Friday, October 25)

1. Griffiths, problem 11.10.
2. Griffiths, problem 11.14.
3. Griffiths, problem 11.21.
4. Griffiths, problem 11.16. (I will give a more detailed discussion of synchrotron radiation later in the course.)
5. Compute the analogue of the Yukawa potential in 2 dimensions. That is, solve, in two-dimensional space,

$$(-\nabla^2 + \mu^2)V = \delta^{(2)}(\vec{x}) \tag{1}$$

   (a) Find a representation of $V(|\vec{x}|)$ as a Fourier transform,

$$V(x) = \int \frac{d^2k}{(2\pi)^2} e^{i\vec{k}\cdot\vec{x}} \; \widetilde{V}(\vec{k}) \; . \tag{2}$$

   (b) Write the integral over $\vec{k}$ in cylindrical coordinates. Integrate over ϕ. We did this integral when we studied the Bessel function $J_0(z)$.

   (c) Extend the integral over k from $-\infty$ to ∞. Note that this is not straightforward, because the Bessel function $J_0(z)$ is an even function of z. You need a sneaky trick, found by rooting around in Abramowitz and Stegun, *Handbook of Mathematical Functions*, Chapter 9. The Hankel function $H_m^{(1)}(z)$ is defined by

$$H_m^{(1)}(z) = J_m(z) + iY_m(z) \tag{3}$$

   where $Y_m(z)$ is the solution of Bessel's equation that is singular at the origin. The Hankel function $H_m^{(1)}(z)$ behaves asymptotically as $e^{iz}/\sqrt{z}$. Because it has a singularity at $z = 0$, it does not reflect the same way as $J_m(z)$. In fact, according to eq. (9.1.39) of Abramowitz and Stegun,

$$H_m^{(1)}(-z) = -(-1)^m \left(J_m(z) - iY_m(z)\right)) \tag{4}$$

   So we can write $V(x)$ as an integral from $-\infty$ to ∞ involving $H_0^{(1)}(z)$. The expression (4) applies when the contour of integration is taken to lie *above* the singularity at $k = 0$. (Why?)

(d) Since $H_0^{(1)}(z)$ has no singularities in the upper half plane and behaves as e^{iz}, we can close the contour in the upper-half plane and eventually contract it onto the pole of $\widetilde{V}(k)$. To pick up the pole, we need to know what $H_0^{(1)}(z)$ looks like for z on the positive imaginary axis. This behavior is described by the modified Bessel function $K_0(z)$; see eq. (9.6.4) and Figure 9.7. Using all of this information, find a final form for $V(x)$ in in terms of $K_0(z)$.

(e) The behavior of $K_0(z)$ near $z = 0$ is given in eq. (9.6.13). Take the limit $\mu \to 0$ and see if you recover the usual electrostatic potential in two dimensions.