Physics 121 – Problem Set # 5

(due Friday, May 17)

1. Consider a string of density ρ and tension κ with a point mass of mass M glued onto it at $z = 0$.

   (a) Write the equation of motion for the displacement $X(t)$ of the mass point. Include the force on the mass point due to the string.

   (b) Find a solution to the wave equation and the equation of the displacement of the mass point in the form:

$$\begin{aligned} \chi(t,z) &= \begin{cases} \mathrm{Re}\,\chi_0\,[e^{ikz-i\omega t} + Re^{-ikz-i\omega t}] & z < 0 \\ \mathrm{Re}\,\chi_0\,[Te^{ikz-i\omega t}] & 0 < z \end{cases} \\ X(t) &= \mathrm{Re} X_0 e^{-i\omega t} \end{aligned} \tag{1}$$

   (c) Show that $|R|^2 + |T|^2 = 1$. Derive this result from general principles. What happens to the mass at late times?

2. Griffiths, problem 9.7.

3. Consider an elastic string with mass density ρ and tension κ. The string is coated in the interval $0 < z < \ell$ so that the mass density increases to ρ_2 in this interval. The tension κ remains constant throughout the string.

   (a) By imposing the condition that both $\chi(z)$ and $\partial\chi/\partial z$ are continuous at both boundaries, find a solution to the wave equation of the form:

$$\chi(t,z) = \begin{cases} \mathrm{Re}\,\chi_0\,[e^{ikz-i\omega t} + Re^{-ikz-i\omega t}] & z < 0 \\ \mathrm{Re}\,\chi_0\,[Ae^{ik_2 z-i\omega t} + Be^{-ik_2 z-i\omega t}] & 0 < z < \ell \\ \mathrm{Re}\,\chi_0\,[Te^{ikz-i\omega t}] & \ell < z \end{cases} . \tag{2}$$

   (b) Show from the explicit solution that $1 = |R|^2 + |T|^2$. Why?

   (c) Plot $|R|^2$ and $|T|^2$ as a function of $(k\ell/2\pi)$ for the case $\rho_2/\rho = 4$. Explain the qualitative behavior. In particular, what happens when $(k\ell/2\pi) = \frac{1}{2}$?

4. Griffiths, problem 9.34 and 9.35. Notice that the prevoius problem was a warm-up for this one.