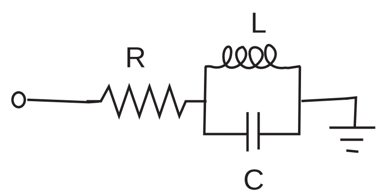
Physics 121 – Midterm Exam

(Friday, May 3)

The exam contains four problems, each democratically worth 25 points.

1. Consider the following circuit:



   (a) Compute the impedance.

   (b) The loop is a resonant circuit with $\omega_0 = 1/\sqrt{LC}$. How is this reflected in the response of the current $I(\omega)$ flowing through the resistor to an applied voltage $V(\omega)$?

   (c) Consider the response of the circuit to a sudden applied voltage $V(t) = V_0\delta(t)$, in the limit of large R, $RC \gg \sqrt{LC}$. With what time constant τ does the oscillating response
   $$I(t) \sim e^{-t/\tau}\cos(\omega t + \alpha) \tag{1}$$
   decay?

2. A capacitor has the form of two parallel circular metal plates of radius 10 cm, placed 1 cm apart. The capacitor is held at 1000 V and then discharged through a 1000 Ω resistor. Assume that the discharge is sufficiently slow that the charge on each plate remains approximately constant over the face of the plate.

   (a) Compute the E field in the capacitor and the stored energy as a function of time. Give the answers in the form
   $$A\exp[-t/a] \tag{2}$$
   with numerical values for A and a.

   (b) Compute the Poynting vector inside the capacitor. Again, give a numerical answer in the form above.

   (c) Using (b), compute the energy flow through a cylinder that encloses the capacitor. Does this flow account for the change in the stored energy?

3. In Physics 124, we will learn that an accelerated charge radiates electromagnetic energy. Then the charge looses kinetic energy. At the (previous) turn of the century, Abraham and Lorentz described this 'radiation reaction' by writing the following equation of motion for a particle in a harmonic oscillator

$$m\frac{d^2}{dt^2}x + m\omega_0^2 x - m\eta\frac{d^3}{dt^3}x = 0 \ , \tag{3}$$

where $\eta = e^2/(6\pi\epsilon_0 c^3)$. In the following, just treat η as a constant much smaller than $1/\omega_0$.

(a) If η is nonzero, the oscillator is damped. If the oscillator has amplitude A_0 at time $t = 0$, find an approximate equation for the amplitude $A(t)$ at time t.

(b) Construct the retarded Green's function for the Abraham-Lorentz equation. To simplify the analysis, use approximations valid when η is small. What is the behavior of the Green's function as $t \to \infty$?

4. Fluids can contain shear stresses. In a fluid with pressure $p(\vec{x})$, velocity $\vec{v}(\vec{x})$, to first order in gradients of $\vec{v}$, the stress tensor is written

$$\sigma^{ij} = -p\delta^{ij} + \mu\left(\frac{\partial v^i}{\partial x^j} + \frac{\partial v^j}{\partial x^i}\right) \tag{4}$$

where the constant μ is called the *viscosity*.

(a) Explain why this is an appropriate form for the stress tensor. What important properties of the stress tensor does it satisfy? Why is can't we add a term involving v^i with no derivatives?

(b) A flat plate is fixed to the ground. A layer of oil is poured onto it, and then another plate is laid on top. The top plate has mass density M and area A. The distance between the plates is h. The viscosity of the oil is μ. Take $\hat{z}$ to be the vertical direction. Assume that, if the top plate moves in the $\hat{x}$ direction, the velocity in the oil is a linear function of z and that the oil just on the boundary of each plate is at rest with respect to the plate. Treat the oil as incompressible. Write the equation of motion for the top plate.

(c) If the top plate is pushed with a force F that is turned on suddenly at time $t = 0$, find the velocity $\dot{x}(t)$.

Some useful quantities:

$$\begin{aligned} \epsilon_0 &= 8.85 \times 10^{-12}\ \mathrm{C^2/Nm^2} \\ \mu_0 &= 4\pi \times 10^{-7}\ \mathrm{N/A^2} \end{aligned} \tag{5}$$