Physics 120 – Problem Set # 4

(due Friday, February 15)

1. Consider the function $f(x) = x$ on the interval $0 < x < 1$. This function can be expanded in a Fourier series:

$$f(x) = \sum_{n=1}^{\infty} a_n \sin n\pi x \tag{1}$$

   (a) Work out a general expression for a_n.

   (b) Let $f_N(x)$ be the approximation to $f(x)$ given by summing the first N terms of (1). Plot $f_N(x)$ for $N = 1, 3, 5, 10$. (Feel free to use any convenient computer package, maybe even the java plotter.)

   (c) Find the number of terms needed to achieve 1% accuracy ($|f_N(x) - f(x)|/f(x) < 0.01$) at $x = 0.1, 0.5, 0.9, 0.95$.

2. Griffiths, problem 3.14.

3. Griffiths, problems 3.48(a). Griffiths gives the answer, so please show your calculations.

4. Consider Laplace's equation in cylindrical coordinates, in which a point $\vec{x}$ is parametrized as $\vec{x} = (r\cos\phi, r\sin\phi, z)$.

   (a) Using the methods described in class, find an expression for the gradiant operator $\vec{\nabla}$ and for the volume element in this coordinate system, and show that

$$-\nabla^2 = -\frac{1}{r}\frac{d}{dr}r\frac{d}{dr} - \frac{1}{r^2}\frac{d^2}{d\phi^2} - \frac{d^2}{dz^2} \tag{2}$$

   (b) Show that the most general solution of Laplace's equation which is independent of z can be written:

$$\varphi(r,\phi) = \sum_m (A_m r^m + B_m r^{-m})e^{im\phi} \tag{3}$$

   where m is summed over integers from $-\infty$ to ∞. The case $m = 0$ is special. Show that in this case the coefficient is $(A_0 + B_0 \log r)$.

   (c) If ϕ is regular inside a cylinder $r < R$, all of the B_m must vanish. Use this observation with the series (2) to find the potential in the situation of Griffiths problem 3.48(b). This is straightforward if you represent the boundary condition at $r = R$ as a Fourier series in ϕ. Then, solve that problem.

5. Consider a capacitor consisting of two thin rectangular metal plates, of dimension $a \times b$, separated by a distance d. Assume, throughout the problem, that $b \gg a, d$, so that the problem is approximately 2-dimensional.

   (a) When $d \ll a$, the capacitance of the capacitor is $C \approx \epsilon_0 ab/d$. Find an approximate expression for the capacitance when $a \ll d \ll b$.

   (b) Using dimensional analysis, show that the general expression for the capacitance when b is large has the form

   $$C = g(d/a) \cdot \epsilon_0 ab/d \tag{4}$$

   where $g(x)$ is a dimensionless function to be determined. Show that, as $x \to 0$, $g(x) \to 1$, and find the behavior of $g(x)$ as $x \to \infty$ using the result of part (a).

   (c) Using the java applet of the previous problem set, compute the capacitance of three 2-dimensional capacitors with varying ratio of a to d. The easiest way to compute the capacitance is to use the fact that the energy stored in a capacitor is $E = \frac{1}{2}CV^2$ and then use the energy calculation from the applet. Determine the values of $g(x)$ at the three points you have chosen.

   (d) Sketch a plot of $g(x)$ using the data gathered in this problem.