Physics 120 – Problem Set # 1

(due Friday, January 18)

The purpose of this problem set is to help you review some mathematical methods that will be used in the course. If it turns out that this is not review, and that in fact you have never done these sorts of calculations before, please take special care that you are familiar with these methods.

1. Matrix multiplication: Let

$$M = \begin{pmatrix} 2 & 0 & 3 \\ 5 & 0 & 4 \\ 0 & 6 & 1 \end{pmatrix} \qquad N = \begin{pmatrix} -1 & -3 & 4 \\ 2 & 2 & 3 \\ 7 & 7 & 0 \end{pmatrix} \tag{1}$$

Evaluate

(a)

$$M \begin{pmatrix} 1 \\ 1 \\ 2 \end{pmatrix} \qquad N \begin{pmatrix} -2 \\ 3 \\ 1 \end{pmatrix} \tag{2}$$

(b) MN, NM, $\det M$, $\det N$

(c)

$$\begin{pmatrix} 4 & 0 & -1 \end{pmatrix} M \begin{pmatrix} 2 \\ 0 \\ -2 \end{pmatrix} \tag{3}$$

2. Vector manipulations: Let

$$\vec{a} = (1, 0, 0) \qquad \vec{b} = (0, 3, 7) \qquad \vec{c} = (1, -2, 2) \tag{4}$$

Evaluate:

(a) $\vec{a} \cdot \vec{b}$, $\vec{a} \cdot \vec{c}$, $\vec{b} \cdot \vec{c}$.

(b) $\vec{a} \times \vec{b}$, $\vec{a} \times \vec{c}$, $\vec{b} \times \vec{c}$

3. Vector rotations: Using the same vectors $\vec{a}$, $\vec{b}$, $\vec{c}$ as in Problem 1,

(a) Evaluate, in any convenient way, the result of rotating each of these vectors by $45°$ in the positive sense about the $\hat{y}$ axis.

(b) Evaluate, in any convenient way, the result of rotating each of these vectors by $60°$ in the negative sense about the $\hat{z}$ axis.

(c) Rewrite each operation as a matrix action. That is, write 3×3 matrix R_a such that the answer to part (a) is $R_a\vec{a}$, $R_a\vec{b}$, $R_a\vec{c}$, and similarly find the matrix R_b for part (b).

(d) Find the matrix that represents a rotation by $45°$ about the $\hat{y}$ axis, followed by a rotation by $-60°$ about the $\hat{z}$ axis. Also, find the matrix that represents a a rotation by $-60°$ about the $\hat{z}$ axis, followed by a rotation by $45°$ about the $\hat{y}$ axis. Should these be equal?

4. Numerical integration of differential equations:

(a) In a separate handout, you will find instructions for the use of a java applet plotter called `myPlotter`. To warm up, use `myPlotter` to make plots of the following functions:

$$y = 3 + 2x \ , \qquad y = x^3 - x \tag{5}$$

(b) Use `myPlotter` to draw a circle of radius 2. Remember that the sin and cos functions are written in java as `Math.sin`, `Math.cos`, with the argument in radians.

(c) Show analytically that this circle is the result of integrating the differential equations

$$\dot{x} = -y \ , \qquad \dot{y} = x \tag{6}$$

with the initial condition $x(0) = 2$, $y(0) = 0$.

(d) Approximate these equations by the discrete equations

$$x_{n+1} = x_n - \Delta t \cdot y_n \ , \qquad y_{n+1} = y_n + \Delta t \cdot x_n \tag{7}$$

and solve these equations iteratively with a rather coarse value of the time step (say, $\Delta t = 0.2$.) You can structure the numerical solution in java as a `for` loop with t as the stepping variable. Plot the result on top of the exact circle determined in part (b).

(e) Approximate the equations in (c) by the discrete equations

$$x_{n+1} = x_n - \Delta t \cdot y_n \ , \qquad y_{n+1} = y_n + \Delta t \cdot x_{n+1} \tag{8}$$

and solve these equations iteratively with the same coarse value of the time step (say, $\Delta t = 0.2$.) Plot the result on top of the exact circle determined in part (b).

(f) Explain the failures of the two approximations. Do the deviations become smaller when the time step is decreased? Is there a discretization that gives a better approximation when Δt is large?