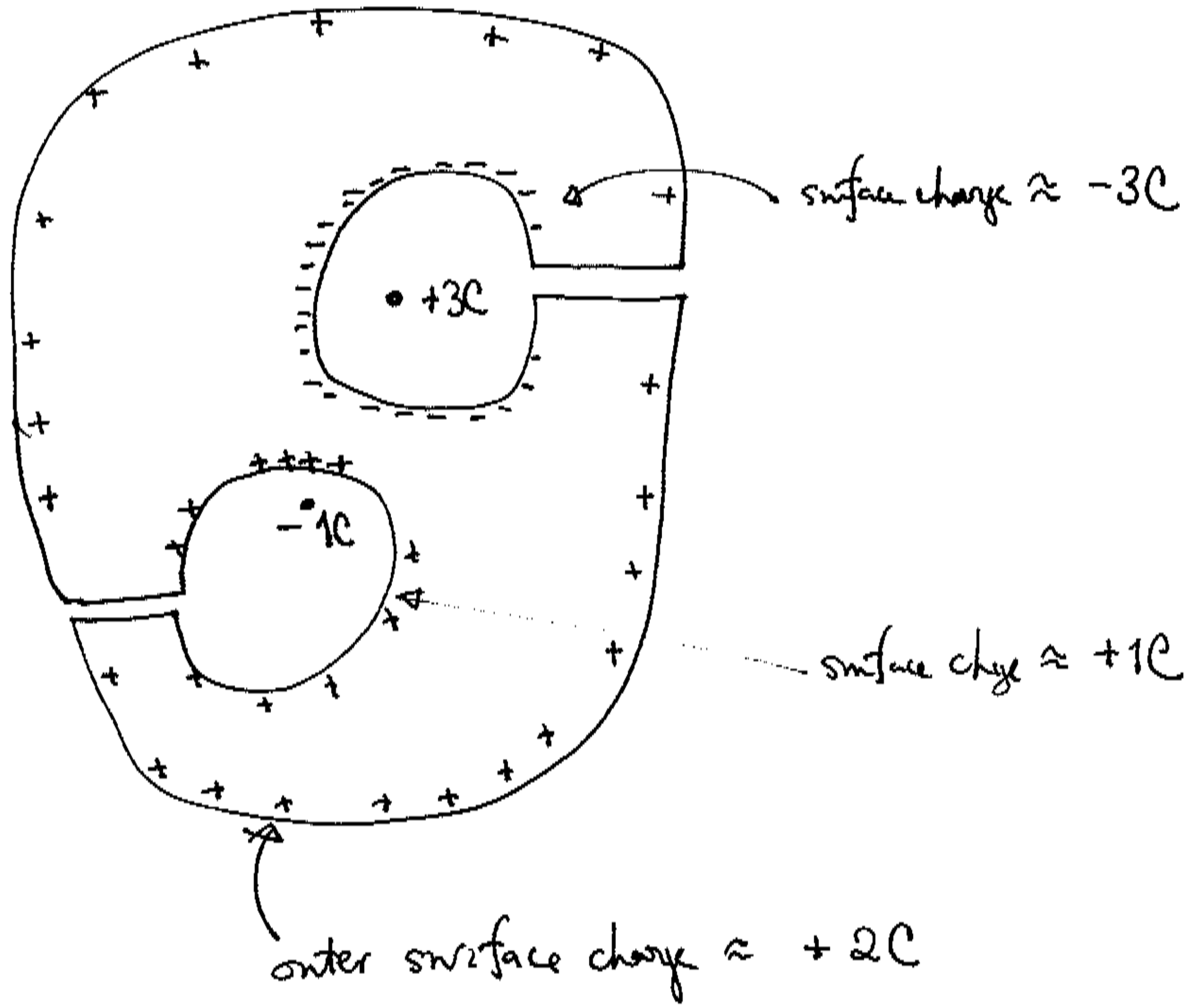
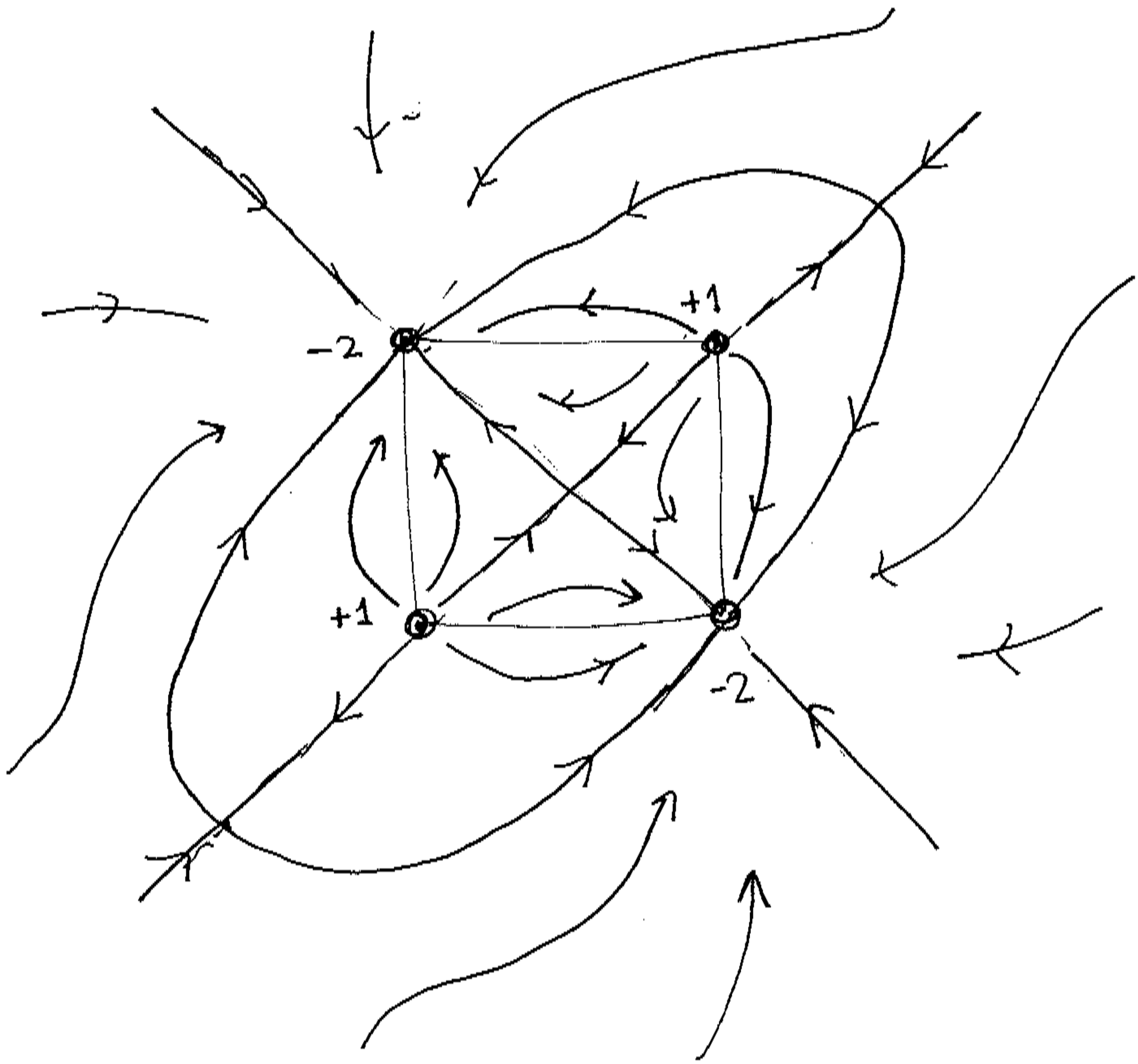
Physics 120 – Midterm – Answers

1. The conductor remains electrically neutral.
Also, inside the bulk of the conductor, $\vec{E}=0$
So, the external charge is neutralized around each hole:

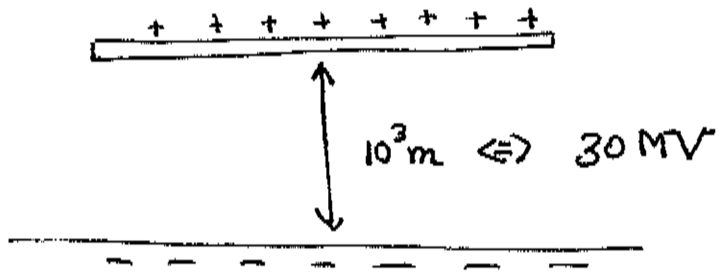


total charge on the conductor = 0

2.) Half of the flux flowing into the −2 charges comes from the +1 charges. The other half comes from ∞



3.



a.) The $\vec{E}$ field in this "capacitor"

is $30\,MV / 10^3 m = 3 \times 10^4\ V/m$

$J = Nm$

$$\rho = \varepsilon_0 \cdot E = 9 \times 10^{-12}\ C^2/Nm^2 \cdot 3 \times 10^4\ J/Cm$$

$$= 2.7 \times 10^{-7}\ C/m^2$$

$$\rho = 3 \times 10^{-7}\ C/m^2 = 0.3\ C/km^2$$

b.) The energy stored is $\qquad$ $Vol. = 1km \cdot (1km)^2 = 10^9 m$

$$E = \tfrac{1}{2}\epsilon_0 (\vec{E})^2 \cdot Vol.$$

$$= \tfrac{1}{2}\,(9 \times 10^{-12}\ C^2/Jm)\,(3 \times 10^4\ J/Cm)^2 \cdot (10^9 m^3)$$

$$= \tfrac{1}{2} \cdot 81 \times 10^5\ J$$

$$E = 4 \times 10^6\ J$$

c.) $I = V/R = 3 \times 10^7\ V / 10^{-3}\Omega = 3 \times 10^{10}\ A$

[Apologies, I should have written $R = 10^3\,\Omega$; then
$I = 3 \times 10^4\ A$.]

d.) Roughly, the discharge takes

$$\text{charge / initial current} = 0.3\ C/\text{km}^2 \ / \ 3\times10^{10}\ A$$

$$= 10^{-11}\ \text{sec}$$

[for the correct resistance $\tau = 0.3\ C/\text{km}^2 \ / \ 3\times10^{4}\ A$

$= 10^{-5}$ sec]

The discharge of a capacitor is an exponential decay with a time constant

$$\tau = RC = (Q/V)\cdot R = Q/(V/R)$$

and V/R is the initial current, so the time constant of the exponential decay is given by the time above.

For a more complete story on lightning, see the Feynman lectures, vol. II, lecture

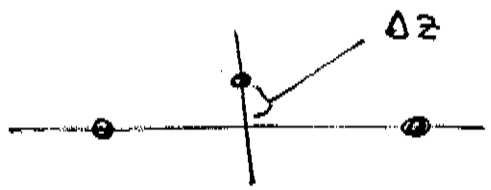
4.)

the force/m on the middle wire is

$$\vec{F} = \rho \cdot \frac{1}{2\pi\epsilon_0}\frac{\rho}{a}(-1,0) + \rho\frac{1}{2\pi\epsilon_0}\frac{\rho}{a}(1,0)$$

$$= 0$$

a.) Lift the middle wire up by Δz



now the force/m is

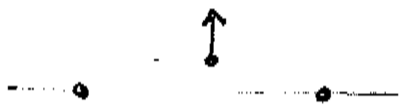
$$\vec{F} = \frac{\rho^2}{2\pi\epsilon_0}\frac{1}{(a^2+\Delta z^2)}(-a, \Delta z)$$

$$+ \frac{\rho^2}{2\pi\epsilon_0}\frac{1}{[a^2+(\Delta z)^2]}(a, \Delta z)$$

$$= \frac{\rho^2}{2\pi\epsilon_0}\frac{1}{[a^2+(\Delta z)^2]}(0, 2\Delta z)$$

or

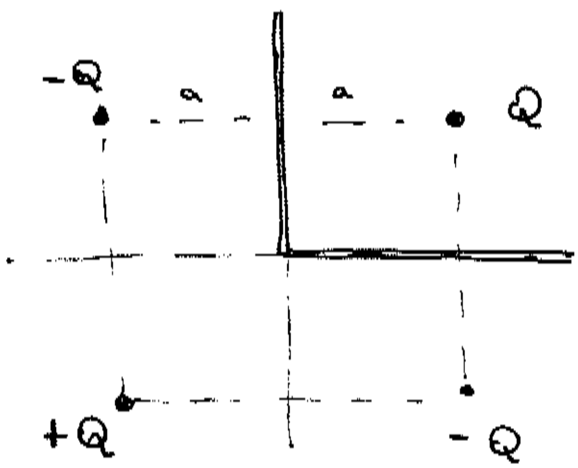
$$\text{Force}/m = \frac{\rho^2}{\pi \epsilon_0} \frac{\Delta z}{a^2} \hat{z}$$



b) Since the force points away from $\Delta z = 0$, the equilibrium is unstable, as required by Earnshaw's thm.

5.)

a.) The following set of image charges gives a solution to Poisson's equation with $\phi = 0$ on the conducting planes



The potential in the physical region is

$$\phi = \frac{Q}{4\pi\epsilon_0}\left(\frac{1}{|\vec{r}-\vec{A}|} - \frac{1}{|\vec{r}-\vec{B}|} - \frac{1}{|\vec{r}-\vec{C}|} + \frac{1}{|\vec{r}-\vec{D}|}\right)$$

$\vec{A} = (a, a, 0)$

$\vec{B} = (-a, a, 0)$

$\vec{C} = (a, -a, 0)$

$\vec{D} = (-a, -a, 0)$

b.) the force on the charge is

$$\vec{F} = \frac{Q^2}{4\pi\epsilon_0} \left\{ \frac{(-1,0)}{(2a)^2} + \frac{(0,-1)}{(2a)^2} + \frac{\frac{1}{\sqrt{2}}(1,1)}{(2\sqrt{2}a)^2} \right\}$$

$$= \frac{Q^2}{4\pi\epsilon_0} \frac{1}{a^2} \cdot \left(\frac{1}{4} - \frac{1}{8\sqrt{2}} \right) (-1,-1)$$

c.) To find the work needed to bring in the charge from ∞, we can procede in two ways.

① Integrate the force from ∞ :

$$W = - \int d\vec{x} \cdot \vec{F}$$

$$= - \int_a^\infty dx \frac{Q^2}{4\pi\epsilon_0 x^2} \left(\frac{1}{4} - \frac{1}{8\sqrt{2}} \right)$$

$$- \int_a^\infty dy \frac{Q^2}{4\pi\epsilon_0 y^2} \left(\frac{1}{4} - \frac{1}{8\sqrt{2}} \right)$$

$$= - 2 \frac{Q^2}{4\pi\epsilon_0 a} \left(\frac{1}{4} - \frac{1}{8\sqrt{2}} \right)$$

So

$$W = -\frac{Q^2}{4\pi\epsilon_0 a}\left(\frac{1}{2} - \frac{1}{4\sqrt{2}}\right)$$

W should be negative, because the charge is attracted to the conductors.

(2) Compute the final energy of the whole charge distribution. The charge on the conductor is at $\phi = 0$, so this does not contribute to $\int d^3x\, \rho \cdot \phi$. So we use the charge at (a, a):

$$\text{Energy} = \frac{1}{2} Q \cdot \phi$$

$$= \frac{1}{2} Q \left(+ \frac{Q}{4\pi\epsilon_0}\left(-\frac{1}{2a} - \frac{1}{2a} + \frac{1}{2\sqrt{2}a}\right)\right)$$

$$= -\frac{Q^2}{4\pi\epsilon_0 a}\left(\frac{1}{2} - \frac{1}{4\sqrt{2}a}\right)$$

in agreement w. (1).

Notice ~~that~~

Energy $= Q \cdot \phi$ is not correct, since the charge on the conductors moves as we bring the external charge in.

∇