Physics 120

Intermediate Electricity and Magnetism

Syllabus:

Electrostatics

Laplace's equation

Electric fields in matter

Magnetostatics and magnetic materials

Faraday's laws

Physics 121:

Maxwell's equations, dynamic EM fields

Physics 122:

Electromagnetic radiation

References:

textbook:

Griffiths: Introduction to Electrodynamics

and Reitz and Milford: Foundations of Electromagnetic Theory

Heald and Marion: Classical Electromagnetic Radiation

Feynman, Leighton, and Sands, The Feynman Lectures on Physics, vol. 2

the bible for advanced students:

Jackson, Classical Electrodynamics

and, sometime during the year, take time to read:

Feynman, QED, the Strange Theory of Matter and Light.

Physics 120 - 121 - 122 is the foundational undergraduate course in the theory of electricity and magnetism. The effects of electromagnetism are all around us: Electromagnetic forces determine the structure of atoms and molecules and thus of all of the materials we encounter. Light is electromagnetic in nature, as are the many forms of radiation on both sides of the visible spectrum. So understanding electromagnetism will take us a long way in understanding how Nature is built up. But electromagnetism is important not only for facts about its structure but also conceptually. The study of electromagnetism has provided methods for analyzing a large number of physical systems, including fluids and other media that support flows and waves. And, electromagnetism has proved to be the model for building successful theories of the other fundamental interactions of Nature — the strong, weak, and even gravitational interactions.

In this course, we will develop electromagnetic theory along three dimensions. The first major theme of the course will be the development of a field or local viewpoint on the fundamental interactions. Newton built a model of the cosmos based on action at a distance —

forces between particles act across space. The electric forces were originally described in the same way, as forces between particles which carry electric charge. In the nineteenth century — and especially with the insights of Michael Faraday — this viewpoint changed to one in which charged particles set up fields that fill space. Other particles feel the fields where they are — locally — and react accordingly. Once we admit the reality of electric and magnetic fields that fill space, these fields can take on their own dynamics. They can carry energy and momentum and transport these from place to place. So we will see the progression:

particle interactions → static fields → dynamic fields

→ fields in motion, carrying energy and momentum

Today, we believe that all particles in Nature are associated with fields in motion. The discrete nature of particles is the result of applying the rules of quantum mechanics (you study for next year) to the field dynamics.

The second major theme of the course will be the analysis of linear systems. Electromagnetic interactions are linear to a very good approximation: twice the charge gives twice the field, and twice the force. Linear systems are enormously simpler than systems with non-linear response, and many mathematical methods have been developed specifically for linear equations. Few other systems in Nature are exactly linear, but many systems are approximately linear and benefit from the use of these methods. So we will develop the basic methods for linear systems:

systems with explicit sources → systems with implicit sources
(boundary conditions)

→ dynamic systems

→ systems with wave motion, scattering, diffraction

Finally, we will study many concrete physical systems in which electromagnetic interactions are important. Here we will progress from simple idealized systems to more complex ones:

simple arrays of charges → more realistic materials

→ complex geometries

and systems with wave interactions.

To begin, consider the simplest manifestation of electromagnetism, the interaction of static electric charges. Coulomb and others showed that particles with electric charge interact through inverse square forces proportional to the quantity of charge on each particle. For two particles

$$(\overrightarrow{\text{Force of 1 on 2}}) = \frac{1}{4\pi\epsilon_0} \frac{Q_1 Q_2}{(r_{21})^2} \hat{r}_{21}$$

where Q_1, Q_2 are the charges of 1 and 2 and $\vec{r}_{21} = \vec{r}_2 - \vec{r}_1$. Let me introduce some notation that I will use throughout the course:

$\vec{r}$ is a vector

$\hat{r}$ is a unit vector in the direction of $\vec{r}$

r or $|\vec{r}|$ is the length of $\vec{r}$

$$\left(\hat{r} = \frac{\vec{r}}{|\vec{r}|} \right)$$

Today we know that electric charge is quantized in units of the charge carried by an electron. The electric charge of a proton is the same to high accuracy:

$$\left| \frac{Q_p + Q_e}{Q_e} \right| < 1.0 \times 10^{-21} \qquad !$$

However, historically, electric charge is measured in units called coulombs:

$$e = Q_p = -Q_e = 1.602176 \times 10^{-19}\ C$$

The factor $\frac{1}{4\pi\epsilon_0}$ in the formula for the electric force is the conversion factor needed to express the force due to the arbitrary quantity of charge represented by 1 C ($\sim 6\times10^{18} e$) in newtons:

$$\frac{1}{4\pi\epsilon_0} = 8.988 \times 10^{9}\ N \cdot \frac{m^2}{C^2}$$

or

$$\epsilon_0 = 8.85419 \times 10^{-12}\ \frac{C^2}{N \cdot m^2}$$

I very much prefer working in units in which $\epsilon_0 = 1$ ("rationalized Heaviside units"), but this makes it almost impossible to talk to engineers. So in Physics 120-121-122 we will use SI units and carry ϵ_0 in our formulae.

If $Q_1\ Q_2 \cdots Q_N$ are the charges of particles at $\vec{r}_1\ \vec{r}_2 \cdots \vec{r}_N$, the force on a particle of charge Q_0 at $\vec{r}_0$ is

$$\vec{F} = \frac{1}{4\pi\epsilon_0}\frac{Q_0 Q_1}{|\vec{r}_0-\vec{r}_1|^2}(\widehat{r_0-r_1}) + \frac{1}{4\pi\epsilon_0}\frac{Q_0 Q_2}{|\vec{r}_0-\vec{r}_2|}(\widehat{r_0-r_2}) + \cdots$$

or $$\vec{F} = \sum_i \frac{1}{4\pi\epsilon_0} \frac{Q_0 Q_i}{|\vec{r}_{oi}|^2} \hat{r}_{oi}$$

The total force is the vector sum of the forces due to the individual particle interactions. This is the "principle of superposition", a manifestation of the linear relation of electric forces to source charges. Superposition or linearity should be considered an experimental fact. For particles in vacuum, deviation from this linearity have not be observed directly. From high-energy processes involving light quanta (photons), we can compute that the deviation from linearity under ordinary conditions (~1C at 1m) are at the level of 10^{-40}!

Now we take up Faraday's idea. A particle which feels a force should not have to interrogate all of the other particles which stand at various distances. It should just have to measure something just where it is and act accordingly. So, we write

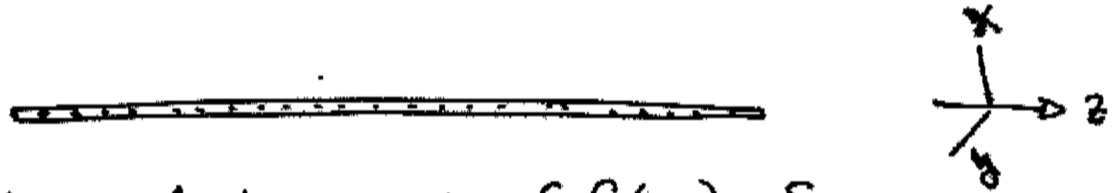
$$\vec{F} = Q_0 \vec{E}(\vec{r}_0)$$

$\vec{E}(\vec{r})$ is the electric field at $\vec{r}$. It is given by

$$\vec{E}(r) = \frac{1}{4\pi\epsilon_0}\sum_i \frac{Q_i}{|\vec{r}-\vec{r}_i|^2}\,(\hat{r r_i})$$

where the sum runs over all charged particles in the universe.

To get a feeling for this vector sum, let's evaluate it in some simple situations. Consider a situation in which electrons are distributed uniformly over a straight wire:



with a uniform density of charge ρ (C/m). Since an electron is very small compared to a Coulomb, we can approximate the sum by an integral.

$$\sum_i Q_i = \int_{-\infty}^{\infty} dz\, \rho$$

Take $\vec{r}_i = z\hat{z}$ and $\vec{r} = x\hat{x}$. The symmetry of the problem tells us that, from the value of $\vec{E}$ field at this point, we can infer the value of $\vec{E}$ at any other point by a rotation about $\hat{z}$ and a translation along $\hat{z}$.

$$|r-r_i|^2 = (x^2+z^2)$$

$$(\widehat{r-r_i}) = \left(\frac{x}{\sqrt{x^2+z^2}}\,,\ 0\,,\ \frac{-z}{\sqrt{x^2+z^2}}\right)$$

[Note that this is a unit vector.]

$$\vec{E}(r\hat{x}) = \frac{1}{4\pi\epsilon_0}\int_{-\infty}^{\infty} dz\ \rho\ \left(\frac{x}{[x^2+z^2]^{3/2}}\,,\ 0\,,\ \frac{-z}{(x^2+z^2)^{3/2}}\right)$$

The integral over the z component vanishes by symmetry $z \to -z$.

For the x component, we need

$$\sin\theta = \frac{x}{(x^2+z^2)^{1/2}} \qquad \cos\theta\, d\theta = \frac{-xz\ dz}{[x^2+z^2]^{3/2}}$$

$$\int_{-\infty}^{\infty} dz\ \frac{x^2}{[x^2+z^2]^{3/2}} = \int_0^{\pi} d\theta\ \sin\theta = -\cos\theta\Big|_0^{\pi} = 2$$

so

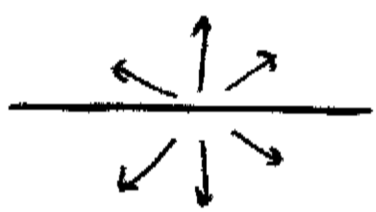
$$\vec{E}(r\hat{x}) = \frac{\rho}{4\pi\epsilon_0}\,\frac{1}{x}\cdot 2\,\hat{x}$$

in general:

$$\vec{E}(\vec{r}) = \frac{\rho}{2\pi\epsilon_0}\,\frac{1}{r_\perp}\,\hat{r}_\perp$$

where if $\vec{r} = (x,y,z)$ $\quad \vec{r}_\perp = (x,y,0)$

This is a cylindrically symmetric field,
falls off as $1/r_\perp$



Similarly, what if charge is distributed uniformly in a plane?

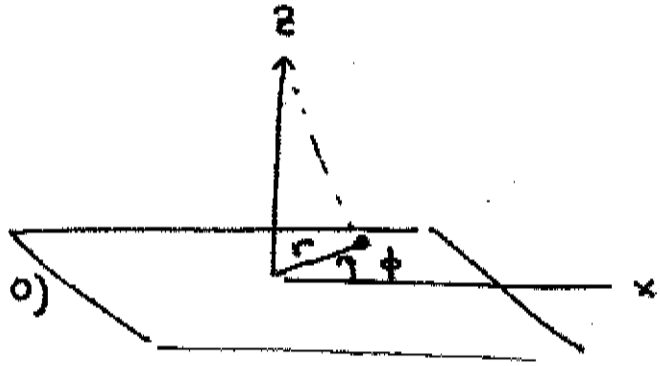
$$\rho = C/m^2 \qquad \sum_i Q_i = \int d^2x \; \rho$$

Set up coordinates

$\vec{r} = (0,0,z)$

$\vec{r}_i = (x,y,0)$

$= (r\cos\phi, r\sin\phi, 0)$



the area element is $\quad d^2x = dr\, r\, d\phi$

$$|\vec{r}-\vec{r}_i|^2 = (r^2+z^2)$$

$$(\widehat{r-r_i}) = \left(\frac{-r\cos\phi}{(r^2+z^2)^{1/2}}, \frac{-r\sin\phi}{(r^2+z^2)^{1/2}}, \frac{z}{(r^2+z^2)^{1/2}} \right)$$

$$\vec{E}(z\hat{z}) = \frac{1}{4\pi\epsilon_0} \int dr\, r\, d\phi \cdot \rho$$

$$\cdot \left(\frac{-r\cos\phi}{(r^2+z^2)^{3/2}}, \frac{-r\sin\phi}{(r^2+z^2)^{3/2}}, \frac{z}{(r^2+z^2)^{3/2}} \right)$$

The integrals $\int_0^{2\pi} d\phi \cos\phi$ and $\int_0^{2\pi} d\phi \sin\phi = 0$

so the $\vec{E}$ field points in the $\hat{z}$ direction, as we might expect.

To evaluate the $\hat{z}$ component, we need:

$$\omega = r^2 \quad d\omega = 2r\,dr$$

$$\int_0^\infty dr\; r \frac{z}{(r^2+z^2)^{3/2}} = \frac{1}{2}\int_0^\infty d\omega \frac{z}{(\omega+z^2)^{3/2}}$$

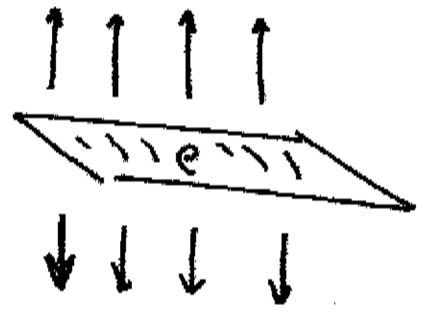
$$= \frac{1}{2}\cdot\left(-\frac{2z}{[\omega+z^2]^{1/2}}\right)\Bigg|_0^\infty = \frac{z}{(z^2)^{1/2}} = 1$$

so

$$\vec{E}(z\hat{z}) = \frac{\rho}{4\pi\epsilon_0}\cdot 1 \cdot \underbrace{2\pi}_{\int_0^{2\pi} d\phi} \cdot \hat{z}$$

i.e.

$$\vec{E} = \frac{\rho}{2\epsilon_0}\hat{z} \qquad \text{outward, independent of distance.}$$



It was a lot of work to obtain these results from vector superposition. We'll discuss a better method next time.