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September 7-10, 2015

The Mystery of Electroweak Symmetry Breaking

4. Composite Higgs fields

Supersymmetry is only one avenue toward a dynamical theory of EWSB. In the previous lecture, I hinted thata we could also forbid quadratic divergences in the Higgs mass by relating the Higgs field to a gauge field through a symmetry

$$\delta\varphi = \epsilon^m A_m$$

The mechanism of fermion pair condensation that I discussed in the second lecture was also appealing. In any such scheme, the Higgs boson is not an elementary particle but rather is a fermion-antifermion bound state. Any radiative corrections to the Higgs mass are cut off at the size scale of the bound state. The specific realization of the pair condensation mechanism that I described did not work, but there may be other ways to use this idea. In this lecture, I will pick up both of these strands and discuss models in which the Higgs field is composite.

To begin, consider the idea of fermion pair condensation more broadly. A model with pair condensation has massless fermions transforming under a chiral symmetry G that is spontaneously broken to a subgroup H. In the second lecture, we assumed that this breaking was directly responsible for $SU(2) \times U(1)$ breaking. In our explicit example of technicolor, we had

$$G = U(1) \times SU(2) \times SU(2)$$

and

$$H = U(1) \times SU(2)$$

G contained the $SU(2) \times U(1)$ gauge symmetry but H contained only the $U(1)$ symmetry of electromagnetism.

Another possibility is to choose a model in which H can contain all of $SU(2)\times U(1)$, so that the pair condensation does not break the electroweak symmetry. In this context, we can arrange that the broken generators of G contain a multiplet that transforms as a doublet under $SU(2)$. Then we will have Goldstone bosons with the quantum numbers of the Higgs doublet field. These bosons will be fermion-antifermion composites. I will show that we can generate a potential for these particles at a second stage, giving EWSB with a light composite scalar particle. Models of this type, in which the Higgs doublet arises as a multiplet of Goldstone bosons, are called *Little Higgs* models.

A simple example is given by a model with the chiral symmetry breaking pattern

$$U(1) \times SU_L(3) \times SU_R(3) \to U(1) \times SU(3)$$

corresponding to 3 flavors of techniquarks. There are 8 Goldstone bosons, corresponding to an $SU(3)$ octet of particles. We can write the corresponding fields in the form of an $SU(3)$ matrix,

$$\Pi^a t^a = \left(\begin{array}{c|c} -\frac{1}{2}\Phi_0 + \vec{\Phi}\cdot\vec{\sigma}/2 & \varphi \\ \hline \varphi^\dagger & \Phi_0 \end{array} \right) \qquad \mathrm{tr}[t^a t^b] = \frac{1}{2}\delta^{ab}$$

Typical hadrons in this strong interaction theory have mass $M \sim 4\pi F$. However, the 8 Goldstone bosons are masses as long as the $SU(3) \times SU(3)$ chiral symmetry remains exact.

The multiplet φ in the representation above is a complex doublet, and we can try to identify it as the Higgs field. If the chiral symmetry is weakly broken, a nonzero potential for φ may be generated by radiative corrections. If these orrections give φ a negative $(\text{mass})^2$, we will have a dynamical theory of EWSB.

A multiplet of Goldstone bosons is described phenomenologically by a nonlinear sigma model. We characterized the vacuum state with pair condensation as

$$\langle Q_{La}^{\dagger} Q_{Rb} \rangle = -\Delta \delta_{ab}$$

But there are equivalent vacuum states found by acting with chiral $SU(3)] \times SU(3)$ transformations

$$Q_L \to V_L Q_L \qquad Q_R \to V_R Q_R$$

These sweep out a space of vacuum states

$$\langle Q_{La}^{\dagger} Q_{Rb} \rangle = -\Delta U_{ab}$$

$$(U = \text{unitary matrix})$$

with chiral $SU(3) \times SU(3)$ acting on these by

$$U \to V_L^{\dagger} U V_R^{T}$$

The Goldstone bosons are local fluctuations in the vacuum orientation. To represent them, write

$$\langle Q_L^{\dagger} Q_R \rangle = -\Delta U(x)$$

with

$$U(x) = \exp[\, i \Pi^a(x) t^a / F \,]$$

The constant F will later be identified with the pion decay constant. The reason to use $U(x)$ rather than $\Pi^a(x)$ is that $U(x)$ has a simpler transformation law under the chiral symmetries,

$$U(x) \to V_L^\dagger U(x) V_R^T$$

A Lagragian for the Goldstone bosons is

$$\mathcal{L} = F^2 \, \mathrm{tr}[\, \partial^m U^\dagger \partial_m U \,]$$

This Lagrangian is $SU(3) \times SU(3)$ invariant. To quadratic order,

$$\mathcal{L} = \frac{1}{2} (\partial_m \Pi^a)^2 + \cdots$$

Notice that there is no way to generate an $SU(3) \times SU(3)$-invariant term with no derivatives. For example,

$$\mathrm{tr}\, U^\dagger U = \mathrm{tr}[1] = \text{indep. of } \Pi^a$$

This makes it impossible to generate a mass for the Π^a. This is a nice implementation of Goldstone's theorem.

We next want to couple Π^a to the weak interaction $SU(2)$. To do this, we gauge an $SU(2)$ subgroup of the unbroken $SU(3)$,

$$\delta U = i\alpha^a \begin{pmatrix} \sigma^a/2 & \\ & 0 \end{pmatrix} U - i\alpha^a U \begin{pmatrix} \sigma^a/2 & \\ & 0 \end{pmatrix}$$

If $U = 1$, $\delta U = 0$ and so $SU(2)$ is not spontaneously broken. The kinetic term for U is modified to

$$\mathcal{L} = F^2 \operatorname{tr}\left[D^m U^\dagger D_m U\right]$$

where

$$D_m U = \partial_m U - ig A_m^a \begin{pmatrix} \sigma^a/2 & \\ & 0 \end{pmatrix} U + ig U \begin{pmatrix} \sigma^a/2 & \\ & 0 \end{pmatrix}$$

You can check that, if $U = 1$, this does not give a mass term for A_m^a. However, if the field that I identified as the Higgs field obtains a vacuum value

$$\Pi^a t^a = \begin{pmatrix} & \begin{matrix} -\frac{i}{\sqrt{2}} v \\ 0 \end{matrix} \\ \begin{matrix} \frac{i}{\sqrt{2}} v & 0 \end{matrix} & \end{pmatrix}$$

then

$$\mathcal{L} = F^2 \operatorname{tr}\left[\left(-ig A_m^a \begin{pmatrix} \sigma^a/2 & \\ & 0 \end{pmatrix}\right)\begin{pmatrix} & \begin{matrix} -\frac{i}{\sqrt{2}} v \end{matrix} \\ & \end{pmatrix}\right)\left(+ig A_m^b \begin{pmatrix} & \\ \begin{matrix} \frac{i v}{\sqrt{2}} & 0 \end{matrix} & \end{pmatrix}\begin{pmatrix} \sigma^b/2 & \\ & 0 \end{pmatrix}\right)\right]$$

$$= F^2 g^2 A_m^a A_m^b \quad \delta^{ab} \frac{1}{4} \cdot \frac{1}{2} \cdot v^2$$

$$= \frac{1}{2} \cdot \frac{g^2 F^2}{4} (A_m^a)^2 = \frac{g^2 F^2}{4}\left[W_m^+ W^{m-} + \ldots\right]$$

and we find

$$m_W = \frac{gF}{2}$$

just as in the second lecture.

The coupling to an $SU(2)$ gauge field breaks the chiral symmetry and allows a Lagrangian term of the form

$$\delta\mathcal{L} = c \cdot g^2 F^4 \, \mathrm{tr}\left[\left[\begin{pmatrix} \sigma^a/2 & \\ & 0 \end{pmatrix}, U^\dagger \right] \left[\begin{pmatrix} \sigma^a/2 & \\ & 0 \end{pmatrix}, U \right] \right]$$

The coefficient of this term has the dimensions $(\text{mass})^4$, and I have given it the mass scale F. This term gives a mass to the diagonal component of Π^a, called $\vec{\Phi}$ above; we find

$$m_\Phi^2 \sim g^2 F^2$$

However, it is interesting that the Higgs field φ does not obtain a mass from this interaction. Since

$$\begin{pmatrix} \sigma^a/2 & \\ & 0 \end{pmatrix} \begin{pmatrix} & \begin{matrix} ih/\sqrt{2} \\ 0 \end{matrix} \\ \begin{matrix} ih/\sqrt{2} & 0 \end{matrix} & \end{pmatrix} \begin{pmatrix} \sigma^a/2 & \\ & 0 \end{pmatrix} = 0$$

the only possible contribution comes from

$$\delta\mathcal{L} \sim c g^2 F^4 \, \mathrm{tr}\left[\begin{pmatrix} \sigma^a/2 & \\ & 0 \end{pmatrix} \begin{pmatrix} \sigma^a/2 & \\ & 0 \end{pmatrix} U U^\dagger \right]$$

which is independent of h. There is a reason for this. Under the action of $SU(2)$, φ transforms under $SU_L(3)$ but not under $SU_R(3)$. Either symmetry is sufficient to prove Goldstone's theorem and require that φ remain massless. This idea, that multiple symmetries can protect Goldstone boson mass and, more generally, the Higgs potential, is called *collective symmetry breaking.* It allows

$$M^2 \sim (10\ \mathrm{TeV})^2 \qquad m_\Phi^2 \sim \alpha_w M^2 \sim (1\ \mathrm{TeV})^2$$

at the first stage of symmetry breaking and

$$m_h^2 \sim (100\ \mathrm{GeV})^2$$

at a later stage. This pushes the strong interaction scale sufficiently high that it does not interfere with precision electroweak predictions.

Here is an example of physics that generates the Higgs potential in a Little Higgs model. To produce the top quark mass, we need terms that couple the Higgs multiplet to the top quark. These can be

$$\delta\mathcal{L} = -y_t F\, \hat{t}_R U_{3i} \chi_{Li} - y_2 F\, \hat{T}_R \hat{T}_L + h.c.$$

I have introduced a multiplet of left-handed quarks

$$\chi_L = \begin{pmatrix} \hat{t} \\ b \\ \hat{T} \end{pmatrix}_L$$

This must be 3-component to preserve $SU_L(3)$. The quark $\hat{T}_L$ must be heavy, so we also need a field $\hat{T}_R$ to allow this to get a large mass. Both $\hat{T}_L$ and $\hat{T}_R$ are singlets under the weak-interaction $SU(2)$.

Notice that the y_1 term preserves $SU_L(3)$ but not $SU_R(3)$. The y_2 term breaks $SU_L(3)$, but this term does not transform under $SU_R(3)$. Both symmetries must be broken to give a mass to φ, so any radiative correction that gives a mass must involve both of these couplings.

Setting $U = 1$, the top quark mass eigenstates are

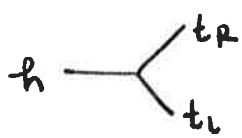
$$T_L = \hat{T}_L \qquad T_R = \frac{y_2 \hat{T}_R + y_1 \hat{t}_R}{\sqrt{y_1^2 + y_2^2}} \qquad m_T = \sqrt{y_1^2 + y_2^2}\, F$$

$$t_L = \hat{t}_L \qquad t_R = \frac{-y_1 \hat{T}_R + y_2 \hat{t}_R}{\sqrt{y_1^2 + y_2^2}} \qquad m_t = 0$$

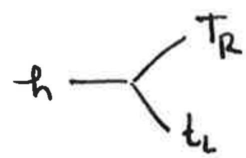
If we also set $\langle h \rangle = v \neq 0$, we also find a mass for the t

$$m_t = \frac{y_t v}{\sqrt{2}} \qquad y_t = \frac{y_1 y_2}{\sqrt{y_1^2 + y_2^2}}$$

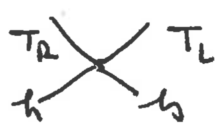
The couplings of the t quarks to the Higgs boson are



$$= -i \frac{y_t}{\sqrt{2}}$$

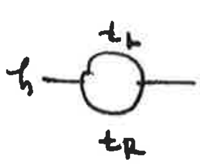


$$= -i \frac{y_1^2}{\sqrt{2}\sqrt{y_1^2 + y_2^2}}$$



$$= +i \frac{y_1^2}{\sqrt{y_1^2 + y_2^2}} \frac{1}{2F}$$

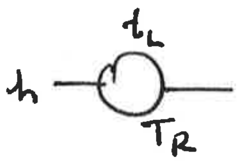
The vertex with two Higgs fields comes from the quadratic term in the expansion of U in terms of Π or h. The 1-loop radiative corrections to the mass of h then come from the three diagrams: a top quark loop, as in the SM,



$$= (-1) \cdot 3 \cdot 2 \cdot \left(-i \frac{y_t}{\sqrt{2}}\right)^2 \int \frac{d^4k}{(2\pi)^4}\, \mathrm{tr}\, \frac{i\sigma \cdot k}{k^2} \frac{i\bar{\sigma} \cdot k}{k^2}$$

$$= -6 \frac{y_1^2 y_2^2}{y_1^2 + y_2^2} \int \frac{d^4k}{(2\pi)^4} \frac{k^2}{k^4}$$

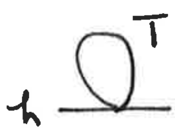
a $t - T$ loop diagram



$$= (-1)\cdot 3\cdot 2\cdot\left(\frac{-i\,y_1^2}{\sqrt{2}\sqrt{y_1^2+y_2^2}}\right)^2\int\frac{d^4k}{(2\pi)^4}\,\mathrm{tr}\left[\frac{i\sigma\cdot k}{k^2-m_T^2}\frac{i\bar\sigma\cdot k}{k^2}\right]$$

$$= -6\,\frac{y_1^4}{y_1^2+y_2^2}\int\frac{d^4k}{(2\pi)^4}\,\frac{k^2}{k^2(k^2-m_T^2)}$$

and a loop diagram using the nonlinear Lagrangian vertex



$$= -3\left(+i\,\frac{y_1^2}{\sqrt{y_1^2+y_2^2}}\right)\frac{1}{2F}\int\frac{d^4k}{(2\pi)^4}\,\mathrm{tr}\,\frac{i\,m_T}{k^2-m_T^2}$$

$$= +6\,\frac{y_1^2}{\sqrt{y_1^2+y_2^2}}\sqrt{y_1^2+y_2^2}\int\frac{d^4k}{(2\pi)^4}\,\frac{1}{k^2-m_T^2}$$

The quadratic divergences cancel among these diagrams. What is left is

$$-6\,\frac{1}{(y_1^2+y_2^2)}\int\frac{d^4k}{(2\pi)^4}\left(\frac{y_1^2y_2^2}{k^2}+\frac{y_1^4}{k^2-m_T^2}-\frac{y_1^2(y_1^2+y_2^2)}{k^2-m_T^2}\right)$$

After Euclidean rotation of the integral, we find

$$= +6\left(\frac{y_1^2y_2^2}{y_1^2+y_2^2}\right)\frac{i}{(4\pi)^2}\int\frac{dk_E^2\,k_E^2\,m_T^2}{k_E^2(k_E^2+m_T^2)}$$

The sign is that of a *negative* mass term for h, just as we obtained from the SM top quark loop contribution. But now the results is not quadratically divergent.

$$\delta\mu^2 = -\frac{3}{8\pi^2}\,y_1^2y_2^2F^2\log\left(\frac{M^2}{m_T^2}\right)$$

The result involves both coupling constants, as required. Putting in $M\sim 10$ TeV, $F = 246$ GeV, we find

$$\delta\mu^2 \cong - y_1^2 y_2^2 \cdot (100 \text{ GeV})^2$$

This is the right order of magnitude, but $y_1, y_2 > y_t \approx 1$, so some cancellation against other effects is required.

As in the case of supersymmetry, partners of the top quark play an essential role in the cancellation of quadratic divergences. These particles must appear, and with masses not far above 1 TeV.

The model that I have just discussed does not account for custodial symmetry. To have custodial symmetry, the group H should include not only $SU(2) \times U(1)$ but also the custodial $SU(2)$ A relatively simple breaking scheme that provides this is

$$G = SU(5) \rightarrow H = SO(5)$$

This model, the "Littlest Higgs" model, is described in Arkani-Hamed, Cohen, Katz, and Nelson, JHEP 0208, 021 (2002) [hep-ph/0206020].

Now we will pick up the other strand, that we can protect the Higgs mass using a symmetry

$$\delta\varphi = \varepsilon^m A_m$$

This is a Lorentz transformation in 5 dimensions, in which the Higgs field is the 5th component of a 5-dimensional gauge field. This origin for the Higgs field in extra dimensions is called *Gauge-Higgs Unification.*

There are many ways to implement this idea. Here I will discuss only a simple model based on the same embedding of $SU(2) \times U(1)$ symmetry in $SU(3)$ as used in the previous example. Now $SU(3)$ will be the gauge symmetry of a 5-dimensional theory on an interval

$$x^5 \in [0, \pi R]$$

bounded by 4-dimensional surfaces ("branes"). It is consistent to explicitly break the gauge symmetry on the branes. I will consider placing boundary conditions on the gauge fields that break $SU(3)$ explicitly, just on the brane, to $SU(2) \times U(1)$.

Write the gauge bosons of $SU(3)$ in the form of a 3×3 $SU(3)$ matrix

$$A_M^A t^A = \left(\begin{array}{c|c} A_M^a \sigma^a/2 - \frac{1}{2} A_M^0 & \varphi_M \\ \hline \varphi_M^\dagger & A_M^0 \end{array} \right)$$

where $M = 0, 1, 2, 3, 5$, $m = 0, 1, 2, 3$, and $A = 1, \ldots, 8$, $a = 1, 2, 3$. At $x^5 = 0$ and $x^5 = \pi R$, place the boundary conditions

$$\partial_5 A_m^a = \partial_5 A_m^0 = 0 \qquad \varphi_m = 0$$

To but a boundary condition on $F_{m5} = \partial_m A_5 - \partial_5 A_m$ consistently with gauge invariance, we need also

$$A_5^a = A_5^0 = 0 \qquad \partial_5 \varphi_5 = 0$$

Then we can solve Maxwell's equations for the plane wave or particle modes of A_M^A. In an appropriate gauge,

$$(-\Box + \partial_5^2) A_m^A = 0$$

so

$$A^a_m = \cos\left(\frac{n x^5}{R}\right) e^{-ik\cdot x} \qquad \varphi_m = \sin\frac{n x^5}{R} \; e^{-ik\cdot x}$$

with k a 4-momentum satisfying

$$k^2 - n^2/R^2 = 0$$

Then n/R is interpreted as the mass of a massive gauge boson in 4-dimensions. The spectrum of the model contains an infinite series of massive gauge bosons, called a *Kaluza-Klein (KK) tower.*

The modes with $n = 0$ (called *zero modes*) represent massless vector bosons. These are the gauge bosons of the construction viewed as a 4-dimensional gauge theory. Notice that the fields φ_m have no zero modes, so the 4-dimensional theory is an $SU(2) \times U(1)$ gauge theory.

The 5th components of the gauge field have the opposite boundary conditions, as given above, so φ_5 has a zero mode. In the 4-dimensional picture, this is a massless scalar that transforms as an $SU(2)$ doublet. We will identify this scalar field as the Higgs field.

As in the case of Little Higgs models, we can generate a potential for the zero mode Higgs field by radiative corrections. As an illustrative example, consider the effect of a 5-dimensional fermion in the 3-dimensional representation of the $SU(3)$ gauge symmetry. I encourage you to also imagine that this fermion carries $SU(3)$ color, and I will refer to it as a 5-dimensional top quark.

The Dirac Lagrangian in 5 dimensions is

$$\bar{\Psi}\,[i\gamma\cdot\partial - m]\,\Psi = 0$$

where γ^M are a set of matrices satisfying

$$\{\gamma^M, \gamma^N\} = 2g^{MN}$$

A representation of these matrices is given by the same matrices that we use in 4 dimensions,

$$\gamma^m = \begin{pmatrix} 0 & \sigma^m \\ \bar{\sigma}^m & 0 \end{pmatrix} \qquad \gamma^5 = \begin{pmatrix} -i & 0 \\ 0 & i \end{pmatrix} \qquad \Gamma = \begin{pmatrix} -1 & \\ & 1 \end{pmatrix}$$

In 4 dimensions, the Lorentz generators have the form

$$\Sigma^{mn} = \frac{-i}{4}[\gamma^m, \gamma^n] = \begin{pmatrix} \sigma^{mn} & \\ & \bar{\sigma}^{mn} \end{pmatrix}$$

so the 4-dimensional Dirac representation splits into the two irreducible representations of left- and right-handed chirality. In 5 dimensions,

$$\Sigma^{m5} = \frac{-i}{4}[\gamma^m, \gamma^5] = \frac{1}{2}\begin{pmatrix} & \sigma^m \\ -\bar{\sigma}^m & \end{pmatrix}$$

so this Lorentz transformation mixes the chirality states.

However, it is still possible to produce chiral fermions from the 5-dimensional theory considered above. Let T_- be a 5-dimensional fermion field. Impose the boundary conditions

$$\Gamma T_- = -T_- \qquad \text{at} \quad x^5 = 0, \pi R$$

The Dirac equation takes the form

$$\left[i\begin{pmatrix} 0 & \sigma\cdot\partial \\ \bar{\sigma}\cdot\partial & 0 \end{pmatrix} + \begin{pmatrix} \partial_5 & 0 \\ 0 & -\partial_5 \end{pmatrix} - m \right] \begin{pmatrix} T_{-L} \\ T_{-R} \end{pmatrix}$$

that is

$$i\bar{\sigma}\cdot\partial\, T_{-L} + (-\partial_5 - m)\, T_{-R} = 0$$

$$i\sigma\cdot\partial\, T_{-R} + (\partial_5 - m)\, T_{-L} = 0$$

For a mode with 4-momentum k, $T \sim \exp[-ik\cdot x]$,

$$\bar{\sigma}\cdot k \quad T_{-L} + (-\partial_5 - m)\, T_{-R} = 0$$

$$\sigma\cdot k \quad T_{-R} + (\partial_5 - m)\, T_{-L} = 0$$

Using $(\overline{\sigma}\cdot k)(\sigma\cdot k) = k^2$,

$$k^2\, T_L = (\partial_5 - m)(\partial_5 - m)\, T_{-L} = 0$$

$$(k^2 - m^2 + \partial_5^2)\, T_{-L} = 0$$

so that

$$T_{-L}, T_{-R} \sim \left[\sin, \cos\left(\frac{n}{R} x^5\right)\right] e^{-ik\cdot x}$$

with

$$k^2 = m^2 + \left(\frac{n}{R}\right)^2$$

These fermion modes appear from the 4-dimensional point of view a a KK tower of massive fermions. The boundary condition that I have suggested is sufficient to solve for both T_{-L} and T_{-R}, since these are related by the Dirac equation.

In the case $m = 0$, there is an $n = 0$ mode with the special structure

$$T_- = \begin{pmatrix} u_L(k) \\ 0 \end{pmatrix} e^{-ik\cdot x} \qquad k^2 = 0$$

Then T_- also contains a massless 4-dimensional fermion which is left-handed chiral. Similarly, a 5-dimensional massless fermion with the boundary condition

$$\Gamma\, T_+ = +\, T_+ \quad \text{at } x^5 = 0, \pi R$$

will contain a right-handed massless fermion plus a KK tower of massive fermions.

For our model with broken $SU(3)$ symmetry, we introduce a massless fermion multiplet in the 3 of $SU(3)$ with the boundary conditions

$$T = \begin{pmatrix} T_- \\ B_- \\ \hline T_+ \end{pmatrix}$$

This multiplet has 3 zero modes, an $SU(2)$ doublet that has left-handed chirality and a singlet with right-handed chirality. It seems that we could pair the fields T_- and T_+ couple two zero modes and form a 4-dimensional massive fermion. However, that coupling is forbidden as long as the $SU(2)$ gauge symmetry remains unbroken.

We can now spontaneously break the $SU(2)$ symmetry by giving a vacuum expectation value to the field φ_5. With this field nonzero, the Dirac equation for T reads

$$\left[i\gamma^m \partial_m + i\gamma^5 \left(\partial_5 - ig \begin{pmatrix} & \varphi_5 \\ \varphi_5^\dagger & \end{pmatrix} \right) \right] T = 0$$

This looks complicated, but we can simplify the equation by removing φ_5 by a gauge transformation. Then if

$$T = \exp\left[i g \int_0^{x^5} dx^{5'} \begin{pmatrix} & \varphi_5 \\ \varphi_5' & \end{pmatrix} \right] T'$$

the field T' satisfies the original Dirac equation

$$\left(i\gamma^m \partial_m + i\gamma^5 \partial_5 \right) T' = 0$$

However, the gauge transformation shifts the boundary condition satisfied by T' at $x^5 = \pi R$. Choose the form of the gauge field expectation value as

$$A_5^A t^A = \left(\begin{array}{cc|c} & & -iv/\sqrt{2} \\ & & 0 \\ \hline iv/\sqrt{2} & 0 & \end{array} \right)$$

Now the boundary conditions on T'_- and T'_+ are

$$T = \begin{pmatrix} c & 0 & s \\ 0 & 1 & 0 \\ -s & 0 & c \end{pmatrix} T'$$

$$(\Gamma + 1)(c T'_- + s T'_+) = 0$$

$$(\Gamma - 1)(-s T'_- + c T'_+) = 0$$

with

$$c = \cos\left(\frac{\pi g R v}{\sqrt{2}} \right) \qquad s = \sin\left(\frac{\pi g R v}{\sqrt{2}} \right)$$

The zero modes are not compatible with these conditions, so the spectrum of T_-, T_+ contains only massive fermion modes. The masses of these fermions form the KK towers

$$m = \left| \frac{n}{R} \pm \frac{\pi g v}{\sqrt{2}} \right| \qquad n = 0, 1, 2, 3$$

It is interesting to compute the contribution of this tower of fermions to the vacuum energy. Fermions contribute a negative amount to the vacuum energy, the energy obtained by filling the Dirac sea

$$V = \sum_{E<0} E$$

Here the states in the Dirac sea have the energies

$$-\left[|\vec{k}|^2 + \left(\frac{\pi g v}{\sqrt{2}} + n \right)^2 \right]^{\frac{1}{2}} \qquad n = -\infty \ldots \infty$$

so

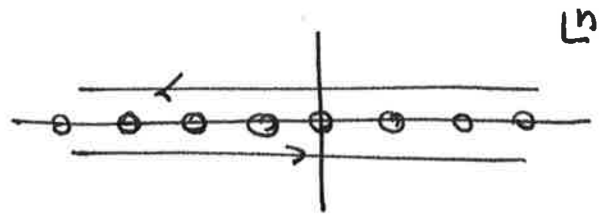
$$V = -\int \frac{d^3k}{(2\pi)^3} \sum_n \left[k^2 + \left(\frac{\pi g v}{\sqrt{2}} + n \right)^2 \right]^{\frac{1}{2}}$$

The sum is divergent. We will cut it off with some regulator such as $\exp[-E^2/\Lambda^2]$ that smoothly suppresses the states at large $|E|$. What I am interested in here is the dependence of the vacuum energy on the Higgs vacuum value v.

To evaluate the sum over n, convert it to an integral

$$\sum_n \left(k^2 + \left(\frac{\pi g v}{\sqrt{2}} + n \right)^2 \right)^{\frac{1}{2}} = \int dn \frac{1}{e^{2\pi i n} - 1} \left[k^2 + \left(n + \frac{\pi g v}{\sqrt{2}} \right)^2 \right]^{\frac{1}{2}}$$

around the contour



The singularities of the integrand are at the points $n =$ integer, and also at the branch points

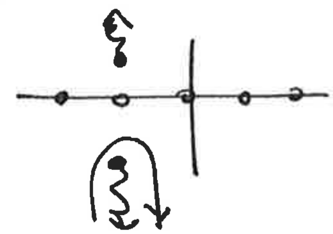
$$n = -\frac{\pi g U}{\sqrt{2}} \pm ik$$

We can evaluate this integral by contour deformation. The lower contour can be pushed to $\mathrm{Im}[n] \to -\infty$, since in this limit we have

$$\frac{1}{e^{2\pi i n}-1} \sim e^{-2\pi |\mathrm{Im}\, n|}$$

The integral catches on the branch cut, and we get a contribution proportional to

$$e^{-2\pi k} \, e^{+2\pi i g U/\sqrt{2}}$$



The upper contour cannot be treated in this way. However, we can write

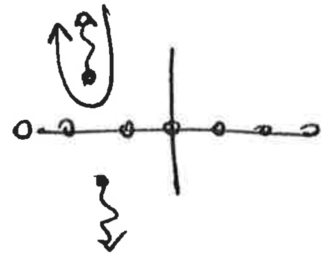
$$\frac{1}{e^{2\pi i n}-1} = -1 - \frac{1}{e^{-2\pi i n}-1}$$

The 1 term gives the integral

$$\int_{-\infty}^{\infty} dn \left[k^2 + \left(\frac{\pi g U}{\sqrt{2}} + n \right)^2 \right]^{\frac{1}{2}}$$

Using our regulator, we can shift the variable n and remove all dependence on v. For the remaining term, we can push the contour up to $\mathrm{Im}[n] \to +\infty$ and find a finite contribution proportional to

$$e^{-2\pi k} e^{-2\pi i g v/\sqrt{2}}$$



In all, we have a divergent term independent of v, and two terms for which the integral over k is convergent. There is a reason that the terms involving v are not divergent. The effect of the gauge field φ_5 is nonlocal. In any small region, φ_5 can be gauged away. The effects of this field are felt only in quantities that are sensitive to the entire 5th dimension. So, φ_5 is intrinsically regulated and does not affect the physics at very short distances.

The complete result for V is

$$V = - \int \frac{d^3k}{(2\pi)^3} \int_k^\infty dy \, (y^2-k^2)^{1/2} \left[\frac{1}{e^{2\pi y - i g v \pi/\sqrt{2}} - 1} + \frac{1}{e^{2\pi y + i g \pi/\sqrt{2}} - 1} \right]$$

plus a v-independent divergent term. The v-dependent term is finite and periodic in v, with a *maximum* at $v = 0$. Thus, this correction produces a potential that drives v to a nonzero value. This is the *Hosotani-Toms mechanism*. It is a dynamical mechanism that operates in a 5-dimensional gauge theory to produce EWSB.

There is a connection betwee the two approaches that I have described in this lecture. Maldacena has proposed a relation called the AdS/CFT correspondence: a conformally invariant theory in d dimensions with global symmetry G has a representation as a theory in $(d+1)$ dimensional anti-de Sitter space with gauge symmetry G. The zero modes of gauge fields in the $(d+1)$-dimensional theory become the gauge bosons of the d-dimensional theory. There is a similar, less rigorous, connection between the two approaches here. The KK states of the $(d+1)$ dimensional theory represent the bound states of a d-dimensional strongly interacting theory.

Indeed, the 5-dimension picture I have described becomes even richer when we replace flat 5-dimensional space by 5-dimensional anti-de Sitter space. This is the construction of Randall and Sundrum. For introductions to the physics of this system, see the reviews by Sundrum, hep-th/0508134, and Contino, arXiv:1005.4269.

The two approaches I have discussed both rely on a spectrum of massive quarks to cancel the ultraviolet divergence of the top quark correction to the Higgs mass. These massive quarks have vectorlike coupling to the weak interactions, so that they can obtain mass without relying on the Higgs vacuum expectation value. Nevertheless, these quarks are similar to heavy versions of the top and bottom quarks, and they can be directly searched for at the LHC. After the 8 TeV run, such heavy vectorlike quarks have been excluded up to about 700 GeV in mass, with the precise value depending on the decay scheme. The ATLAS exclusions are shown in Figure 1. The "naturalness" principle that we applied to SUSY theories might allow these particles to have masses above 1 TeV, but not much higher. The LHC should extend their search for these particles to 1.5 TeV and maybe higher in the current run.

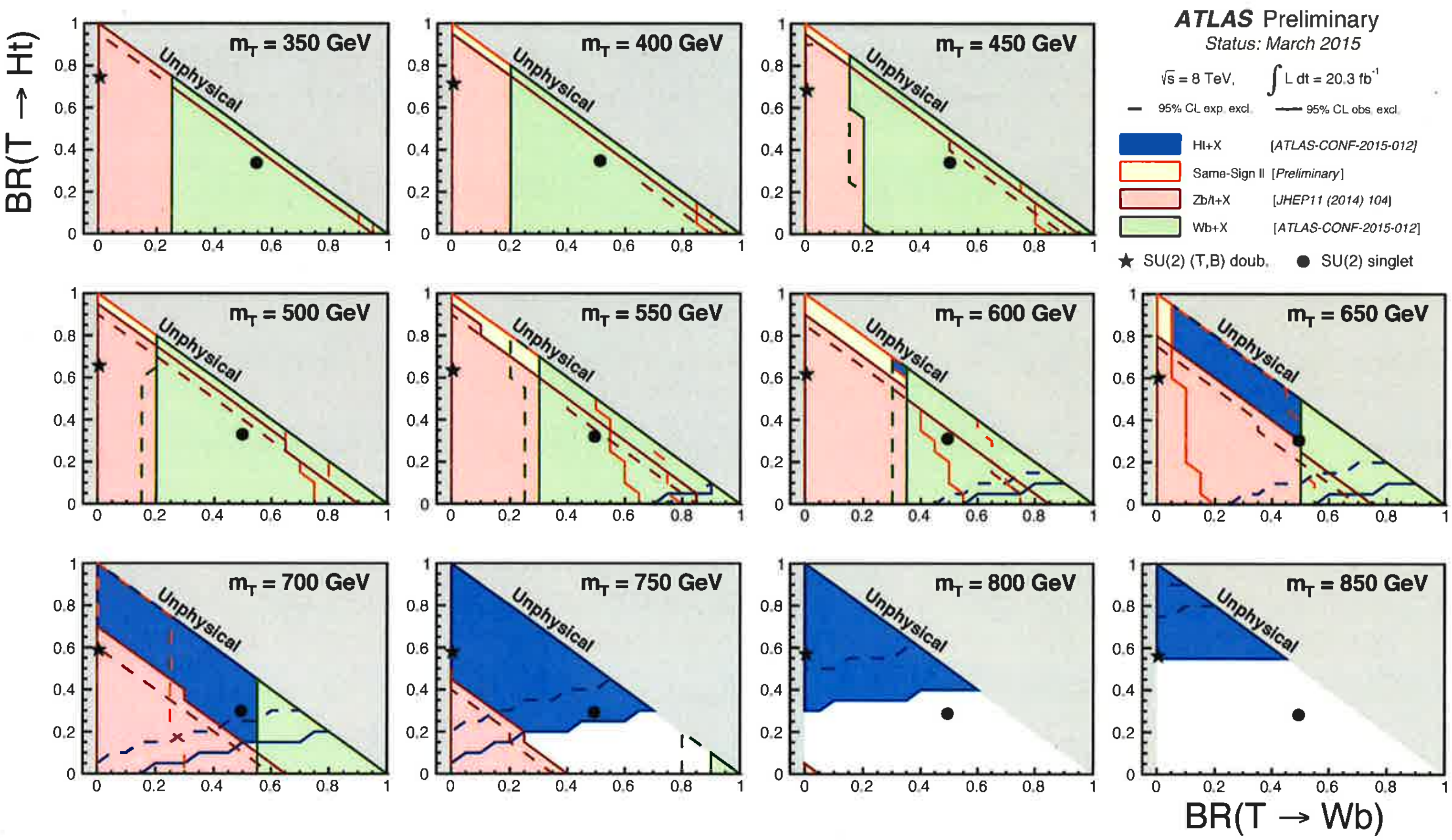
I would like to summarize these lectures with the following points:

1. The SM works well as a theory of elementary particles and, in particular, as a description of EWSB. However, it is only a description, not an explanation.

2. Models that give physics explanations for EWSB are complex. They rely on major new fundamental principles, such as supersymmetry, new strong interactions, or extra space dimensions. Discovering evidence for such a model would bring a new chapter to particle physics.

3. Models that give physics explanations for EWSB are testable in the search for new particles that must appear to cancel the divergences in the SM corrections to the Higgs boson mass. In many different schemes, partners of the top quark play an essential role. It is, to some extent, disappointing that we have already excluded these particles up to rather high masses. But, with so much at stake, can we give up now?

Let's see what the LHC at 13 TeV will bring.



ATLAS limits on $pp \to T\bar{T}$