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The Mystery of Electroweak Symmetry Breaking

3. Supersymmetry

It now seems that we need to develop a model of EWSB that contains an elementary spin 0 $SU(2)$-doublet Higgs field. This model should have the properties that (1) some symmetry forbids the appearance of quadratic divergences in the radiative corrections to the mass of this field, and (2) there is a dynamical calculation that explains the form of the Higgs field potential and the energetics responsible for spontaneous symmetry breaking.

Our pursuit of thi sgoal will lead us into the study of *supersymmetry*, a spacetime symmetry that connects boson and fermion fields. Supersymmetry (SUSY) is a huge and fascinating subject. In these lectures, I will only be able to describe a few aspects of SUSY, from a rather idiosyncratic point of view. If you would like to learn more about supersymmetry, I recommend the following references:

- for an overview: Martin, hep-ph/9709356.
- for the formalism of SUSY: Wess and Bagger, *Supersymmetry and Supergravity*
- for the collider phenomenology of SUSY: Baer and Tata, *Weak Scale Supersymmetry*

My point of view on supersymmetry is discussed further in the TASI lectures arXiv:0801.1928.

We can begin with the question of how to forbid quadratic divergences in a scalar field mass term. We would forbid such divergences if the theory has a symmetry at high energies that forbids the appearance of the operator

$$\delta\mathcal{L} = \mu^2 |\varphi|^2$$

Then additive corrections to this operator cannot be generated by radiative corrections. It is not so easy to find such a symmetry. This operator is $SU(2) \times U(1)$ invariant, CP and P even, and otherwise totally innocuous.

While it is not easy to forbid a mass term for a scalar, we know that there are symmetries that forbid masses for particles of higher spin. Fermion masses are forbidden by the chiral symmetry

$$\psi_L \to e^{i\alpha}\psi_L \qquad \psi_R \to e^{-i\alpha}\psi_R$$

vector boson masses are forbidden by gauge invariance, which is actually required for the construction of a consistent interacting field theory. In both of these cases, masses can arise only by spontaneous symmetry breaking. So we might try to develop models in which there are symmetries that relate scalar fields to spin $\frac{1}{2}$ or spin 1 fields

$$\delta\varphi = \bar{\varepsilon}\psi \qquad \underline{\text{or}} \qquad \delta\phi = \varepsilon^m A_m$$

i will discuss the first alternative in this lecture and the second in the next lecture.

It turns out that trying to extend the first of these transformations to a symmetry of a complete quantum field theory leads to very strong constraints on its structure. Imagine that we have a theory with such a symmetry. Then there is a charge Q_α such that

$$Q_\alpha |\text{boson}\rangle = |\text{fermion}\rangle \qquad Q_\alpha|\text{fermion}\rangle = |\text{boson}\rangle$$

$$[Q_\alpha, H] = 0$$

The charge Q_α must carry spin $\frac{1}{2}$. Without loss of generality, we can assume that Q_α transforms like ψ_L; its conjugate $Q^\dagger_\beta$ transforms like ψ_R. Now consider the anticommutator

$$\{Q_\alpha, Q^\dagger_\beta\}$$

This object is necessarily nonzero if Q is nonzero. It transforms as a 4-vector. So we can represent it as

$$\{Q_\alpha, Q_\beta^\dagger\} = 2\sigma^m_{\alpha\beta} R_m$$

where R_m is a 4-vector operator such that $[R_m, H] = 0$. Now a powerful constraint comes into play. As you know, energy-momentum conservation and Lorentz invariance put powerful constraints on any 2-body scattering amplitude. For fixed spins, the scattering amplitude, a function of the initial and final 4-momenta, can depend only on two combinations of these, the center of mass energy and scattering angle. The new symmetry R_m is as powerful as energy-momentum conservation. If it is a different constraint used in conjunction with the others, scattering is forbidden at a given energy except perhaps at some discrete angles. But, the scattering amplitude is analytic in its arguments, and, from this, it must be zero. Coleman and Mandula converted this argument to a rigorous statement that, if there is any additional 4-vector charge that commutes with the Hamiltonian, the S-matrix of the theory must be zero. Then we can proceed only if R_m is proportional to the total energy-momentum 4-vector of the theory. This gives as the unique form of the anticommutation relation

$$\{Q_\alpha, Q_\beta^\dagger\} = 2\sigma^m_{\alpha\beta} P_m$$

Note that P_m is the total energy-momentum of everything. So, if we introduce supersymmetry as an exact symmetry of the theory, we must supersymmetrize everything—all scalar, fermion, and vector fields and even gravity.

I will now describe the construction of 4-dimensional quantum field theories with SUSY. This construction will depend on detailed properties of 4-dimensional fermions, so I will begin by introducing some appropriate notation. I will write the SUSY transformation on fields using

$$\delta_\xi \Phi = [\xi^T c Q + Q^T c \xi^*, \Phi]$$

where ξ_α is a set of anticommuting numbers (Grassmann numbers) that transform like ψ_L. The symbol c is the matrix

$$c = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} = -i\sigma^2 \qquad c^2 = -1 \qquad c^T = -c$$

If ψ_L is a left-handed fermion

$$\psi_{1L}^T c \, \psi_{2L}$$

is a Lorentz invariant. Also, using the anticommuting property,

$$\psi_{1L}^T c \, \psi_{2L} = + \, \psi_{2L}^T c \, \psi_{1L}$$

Complex or Hermitian conjugation acts on these objects as

$$[\psi_{1L}^T c \, \psi_{2L}]^\dagger = \psi_{2L}^\dagger c^T \psi_{1L}^* = - \psi_{2L}^\dagger c \, \psi_{1L}^* = - \psi_{1L}^\dagger c \, \psi_{2L}^*$$

and

$$[\xi^T c Q]^\dagger = Q^\dagger c^T \xi^* = - Q^\dagger c \, \xi^*$$

If ψ_L transforms as a left-handed fermion,

$$\hat{\psi}_R = - c \, \psi_L^*$$

transforms as a right-handed fermion, so we can rewrite the Dirac Lagrangian

$$\mathcal{L} = \bar{\Psi}(i\gamma\cdot\partial - m)\Psi$$

in terms of only left-handed fields. Let

$$\Psi = \begin{pmatrix} \psi_{1L} \\ -c\psi_{2L}^{*} \end{pmatrix}$$

and

$$\gamma^m = \begin{pmatrix} 0 & \sigma^m \\ \bar{\sigma}^m & 0 \end{pmatrix} \qquad -c\,\sigma^{mT}c = \bar{\sigma}^m$$

Then

$$\mathcal{L} = \psi_{1L}^{\dagger} i\bar{\sigma}\cdot\partial\psi_{1L} + \psi_{2L}^{\dagger} i\bar{\sigma}\cdot\partial\psi_{2L} - m\left[\psi_{1L}^{T}c\psi_{2L} - \psi_{1L}^{\dagger}c\psi_{2L}^{*}\right]$$

Notice that the usual Dirac mass term now takes the form of a Majorana mass and is symmetric with respect to interchange of the two fields.

Now we are ready to write the transformation laws of SUSY. In terms of δ_ξ, and using $P_m = i\partial_m$, the SUSY algebra becomes

$$[\delta_\xi, \delta_\eta] = 2i\left[\xi^{\dagger}\bar{\sigma}^m\eta - \eta^{\dagger}\bar{\sigma}^m\xi\right]\partial_m$$

acting on fields. I will write some specific representations of this algebra. For the argument that these representations are the most general ones possible, see the more careful treatment in Wess and Bagger.

First of all, we need the representation of SUSY that mixes a scalar field ϕ and a fermion field ψ. The formalism should give the same number of fermionic and bosonic degrees of freedom. So we need to relate

$$\text{a complex-valued field } \phi \quad \longleftrightarrow \quad \text{a left-hand chiral field } \psi_L$$

$$\text{particles } \phi \ \ \phi^* \qquad\qquad \text{particles } \psi_L \ \ \bar{\psi}_R$$

Note that the number of fields here is not the same: ϕ is one complex field, and ψ contains two complex (Grassmann) fields. To complete the formalism, we will need to add a second complex scalar field F that does not correspond to any physical particles.

I claim that the following transformation laws satisfy the SUSY algebra

$$\delta_\xi \phi = \sqrt{2}\, \xi^T c\, \psi \qquad \delta_\xi \phi^* = -\sqrt{2}\, \psi^\dagger c\, \xi^*$$

$$\delta_\xi \psi = \sqrt{2}\, i\, \sigma^n c\, \xi^* \partial_n \phi + \sqrt{2} F \xi \qquad \delta_\xi \psi^\dagger = \sqrt{2}\, i\, \xi^T c\, \sigma^n \phi^* + \sqrt{2}\, \xi^\dagger F^*$$

$$\delta_\xi F = -\sqrt{2}\, i\, \xi^\dagger \bar{\sigma}^m \partial_m \psi \qquad \delta_\xi F^* = \sqrt{2}\, i\, \partial_m \psi^\dagger \bar{\sigma}^m \xi$$

The system (ϕ, ψ, F) is called a *chiral supermultiplet.* These transformations leave the following Lagrangian invariant:

$$\mathcal{L} = \partial^m \phi^* \partial_m \phi + \psi^\dagger i \bar{\sigma}\cdot\partial \psi + F^* F$$

Note that the counting of degrees of freedom works out nicely. The new field F indeed has no physical degrees of freedom; it is just a Lagrange multiplier.

To give nontrivial dynamics to this system, we need to add to the Lagrangian some terms without derivatives and with nonlinear interactions. The form of these terms is highly constrained. The unique structure is

$$\mathcal{L}_W = \left(F \frac{dW}{d\phi} - \frac{1}{2} \psi^T c\, \psi \frac{d^2 W}{d\phi^2} \right) + h.c.$$

where $W(\phi)$ is an analytic function of ϕ, called the *superpotential.* If we set

$$W = \frac{1}{2} m \phi^2$$

we obtain

$$\mathcal{L} = \partial^m \phi^* \partial_m \phi + F^* F + m F \phi + m F^* \phi^* + \psi^\dagger i \bar{\sigma}^m \partial_m \psi - m \left[\psi_c^T \psi - \psi_c^\dagger \psi^* \right]$$

The Lagrange multiplier obeys

$$F^* + m\phi = 0$$

Eliminating it, we find

$$\mathcal{L} = \partial^m \phi^* \partial_m \phi - m^2 \phi^* \phi + \psi^\dagger i \bar{\sigma}^m \partial_m \psi - m \left[\psi_c^T \psi - \psi_c^\dagger \psi^* \right]$$

The boson and fermion fields have the same mass m, as required by SUSY.

We can describe the simplest SUSY theory with nonlinear interactions by writing

$$W = \frac{g}{3} \phi^3$$

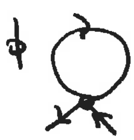
Eliminating F in the same way, we find

$$\mathcal{L} = \partial^m \phi^* \partial_m \phi - g^2 (\phi^* \phi)^2 + \psi^\dagger i \bar{\sigma} \cdot \partial \psi - g \left[\phi \psi_c^T \psi - \phi^* \psi_c^\dagger \psi^* \right]$$

This is a renormalizable field theory with Yukawa and ϕ^4 interactions related in an obvious way. Both fields ϕ and ψ are massless. The mass of ψ is forbidden by an obvious symmetry

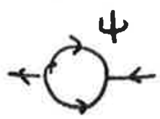
$$\psi \to e^{i\alpha}\psi \qquad \phi \to e^{-2i\alpha}\phi$$

It is not obvious, though, that the mass of ϕ is forbidden. Normally, we would expect a mass for ϕ to be generated by radiative corrections. Let us compute the relevant 1-loop diagrams. These are



$$= (-4ig^2)\int\frac{d^4k}{(2\pi)^4}\frac{i}{k^2} \quad = \quad +4g^2\int\frac{d^4k}{(2\pi)^4}\frac{1}{k^2}$$

and



$$= (-2ig)^2\cdot\frac{1}{2}\cdot(-1)\int\frac{d^4k}{(2\pi)^4}\,\mathrm{tr}\left[\frac{i\sigma\cdot k}{k^2}\,c\,\frac{[i\sigma\cdot(-k)]^T}{k^2}\,c\right]$$

$$= +2g^2\int\frac{d^4k}{(2\pi)^4}\left(-2\frac{k^2}{k^4}\right) = -4g^2\int\frac{d^4k}{(2\pi)^4}\frac{1}{k^2}$$

These diagrams cancel exactly. Similar cancellations appear at all orders of perturbation theory.

This formalism generalizes to theories with many chiral multiplets. The general form of the Lagrangian is

$$\mathcal{L} = \partial^m\phi^{*i}\partial_m\phi^i + \psi^{\dagger i}\,i\bar{\sigma}\cdot\partial\psi^i + F^{*i}F^i$$
$$+\left(F^i\frac{\partial W}{\partial\phi^i} - \frac{1}{2}\psi^{Ti}c\,\psi^j\frac{\partial^2 W}{\partial\phi^i\partial\phi^j}\right) + \text{h.c.}$$

After the elimination of F, the scalar potential has the form

$$V_F = \sum_i \left| \frac{\partial W}{\partial \phi^i} \right|^2$$

Since the ϕ^i are complex and $W(\phi)$ is an analytic function, the equations

$$\frac{\partial W}{\partial \phi^i} = 0$$

normally have a solution on the space of ϕ^i. It can be shown that such a solution gives a global minimum of V_F at zero energy, and that the resulting theory has exact supersymmetry.

It can be shown that the function $W(\phi)$ obtains no additive radiative corrections at any order in perturbation theory. The superpotential can be modified by field strength renormalizations

$$\phi^i \to Z^{ij}(\mu)\, \phi^j$$

but these effects are only logarithmically divergent and cannot add terms of new structure to $W(\phi)$.

Thus, SUSY satisfies our first criterion for a theory of EWSB. If we set the mass μ^2 of the Higgs doublet to zero, it remains zero, to all orders in perturbation theory. There are no additive divergent corrections. The mechanism of the cancellation is that there are new particles whose radiative corrections to μ^2 cancel the corrections from the SM particles. As in the example above, those particlers are the fermionic or bosonic SUSY partners of the SM particles.

To complete the formalism needed to construct a SUSY version of the SM, I need to give the representation of SUSY that contains vector fields. These should be the gauge fields of some gauge group G. Gauge fields belong to *vector supermultiplets.* The content of such a multiplet is real valued vector field A_m $\Longleftrightarrow$ left-hand chiral fermion λ

gauge bosons $A(\varepsilon)$ $\quad$ particles $\lambda_L, \bar{\lambda}_R$

$\varepsilon = L, R$

The field A_m has 3 real bosonic components, not including the gauge degree of freedom. The fermion (gaugino) has 2 complex components, corresponding to 4 real fields. Thus, we need to add one real scalar field, called D, to balance the field content.

The SUSY transformations of a vector multiplet are

$$\delta_\xi A^{ma} = [\xi^\dagger \bar{\sigma}^m \lambda^a + \lambda^\dagger \bar{\sigma}^m \xi] \qquad \sigma^{mn} = \tfrac{1}{2}[\sigma^m \bar{\sigma}^n - \sigma^n \bar{\sigma}^m]$$

$$\delta_\xi \lambda^a = [i\sigma^{mn} F^a_{mn} + D^a \xi]$$

$$\delta_\xi D^a = -i[\xi^\dagger \bar{\sigma}^m \mathcal{D}_m \lambda^a - \mathcal{D}_m \lambda^{\dagger a} \bar{\sigma}^m \xi]$$

where $\mathcal{D}_m$ is the gauge-covariant derivative. These transformations satisfy

$$[\delta_\xi, \delta_\eta] \begin{Bmatrix} A^{ma} \\ \lambda^a \\ D^a \end{Bmatrix} = 2i[\xi^\dagger \bar{\sigma}^n \eta - \eta^\dagger \bar{\sigma}^n \xi] \begin{Bmatrix} F^a_{nm} \\ \mathcal{D}_n \lambda^a \\ \mathcal{D}_n D^a \end{Bmatrix}$$

The right-hand side of this equation is the standard SUSY term with ∂_m plus a gauge transformation. The SUSY transformations of the vector multiplet leave invariant the Lagrangian

$$\mathcal{L} = -\frac{1}{4}(F^a_{mn})^2 + \lambda^{\dagger a} i\bar{\sigma}\cdot\mathcal{D}\lambda^a + \frac{1}{2}(D^a)^2$$

The coupling to matter is described by generalizing the kinetic term of chiral multiplet fields to

$$\mathcal{L} = \mathcal{D}^m\phi^*\mathcal{D}_m\phi + \psi^\dagger i\bar{\sigma}\mathcal{D}\psi + F^*F$$
$$- \sqrt{2}g\left(\phi^*\lambda^{aT}t^a c\psi - \psi^\dagger c\lambda^a t^a\phi\right) + gD^a\left(\phi^* t^a\phi\right)$$

Eliminating D^a generates the D-term potential

$$V_D = \frac{1}{2}\left|\sum_i g\phi^{*i}t^a\phi^i\right|^2$$

So the vector multiplet contains gauge interactions, Yukawa interactions, and ϕ^4 interactions, all governed by the gauge field coupling g. As with the F-term potential, it is typically possible to find solutions to the equations $D^a = 0$, and these give supersymmetric vacuum states.

Now we have all of the ingredients neeed to write a SUSY generalization of the SM. All that we have to do is to put the various SM fields into appropriate supermultiplets. The quark and lepton fields belong to chiral supermultiplets

$$\begin{pmatrix} u_L \\ d_L \end{pmatrix}\begin{pmatrix} \tilde{u}_L \\ \tilde{d}_L \end{pmatrix} \qquad \begin{matrix} \bar{u}_L & \tilde{\bar{u}} \\ \bar{d}_L & \tilde{\bar{d}} \end{matrix} \qquad \begin{pmatrix} \nu_L \\ e_L^- \end{pmatrix}\begin{pmatrix} \tilde{\nu}_L \\ \tilde{e}^- \end{pmatrix} \qquad \left(e_L^+ \quad \tilde{e}^+\right)$$

Notice that I have replaced the right-handed fermions by their left-handed antiparticles. Note also that there are two scalar partners (*squarks or sleptons*) for each quark or lepton, one for each chiral component. The Higgs multiplet similarly must have a fermionic partner, the *higgsino*

$$\varphi \ , \ \begin{pmatrix} \tilde{h}^+ \\ \tilde{h}^0 \end{pmatrix}$$

The gauge fields acquire fermionic partners, *gauginos*

$$(A_m^a, \lambda^a)$$

The Lagrangian of this theory is already almost completely determined. To give the Higgs Yukawa couplings, we must write superpotential terms

$$W = Y_e^{ij}\, \tilde{L}^i * \varphi\, \tilde{\bar{e}}^j + Y_d\, \tilde{q}^i * \varphi\, \tilde{\bar{d}}^j + Y_u\, \tilde{q}^i \cdot \varphi_2\, \tilde{\bar{u}}^j$$

where

$$L * \varphi = L_a \varepsilon_{ab} \varphi_b \qquad L \cdot \varphi = L_a \varphi_a$$

as before. This will also generate 4-scalar interactions among the various fields. It is important to note that one coupling that we used in writing the Yukawa terms of the SM cannot be used here. The coupling

$$Y_u\, q^\dagger * \varphi^* u_R$$

cannot be obtained, because the superpotential must be analytic in fields. So we must introduce a second Higgs doublet, called φ_2 above. From now on, I will call the two Higgs doublets

$$H_d \quad I = \tfrac{1}{2} \quad Y = -\tfrac{1}{2} \qquad H_u \quad I = \tfrac{1}{2} \quad Y = +\tfrac{1}{2}$$

where

$$\langle H_d \rangle = \begin{pmatrix} \frac{1}{\sqrt{2}} v_d \\ 0 \end{pmatrix}$$

gives mass to the d quarks and leptons and

$$\langle H_u \rangle = \begin{pmatrix} 0 \\ \frac{1}{\sqrt{2}} v_u \end{pmatrix}$$

gives mass to the u quarks. With two Higgs doublet fields the mass of the W boson is given by

$$m_W^2 = \frac{1}{4} g^2 \left(v_d^2 + v_u^2 \right)$$

so it is conventional to write

$$v_u = \sin\beta \cdot v \qquad v_d = \cos\beta \cdot v$$

where $v = 246$ GeV as before. Perhaps the explanation of the large size of m_t/m_b is that $\tan\beta$ is large.

If we impose renormalizability and baryon and lepton number conservation, there is only one more term that can be added to the superpotential

$$W = \mu \, H_d * H_u$$

This gives positive (mass)^2 equal to μ^2 for both H_u and H_d. As long as the model is supersymmetric, this mass term receives no additive radiative corrections.

This is a simple and highly constrained theory, but (1) it has no EWSB, and (2) it cannot describe the real world. There is an obvious problem with this theory. SUSY requires the equality of masses for the bosonic and fermionic partners. But there is no boson with the mass of the electron or the u quark. Somehow, SUSY must be spontaneously broken. The spontaneous breaking of SUSY will generate many important effects in the theory, in particular, the forces that drive EWSB.

SUSY is a rather unstable symmetry, in the sense that, if any set of fields, even at high energy, spontaneously break this symmetry, the symmetry breaking can be communicated to all other fields in the theory. It makes sense, then, to think about SUSY breaking in a sector at very large mass scales. This symmetry-breaking induces SUSY-breaking terms for lighter fields, including the SM fields. These can be of the form of SUSY-breaking mass terms

$$\delta\mathcal{L} = -\sum_i m_i |\phi_i|^2 - \sum_a m_a \left[\lambda^{Ta} c \lambda^a + h.c.\right]$$

and cubic interactions among the scalar fields (called *A terms*). These interactions have the common property that they are *soft*, that is, of dimension less than 4. These operators do not generate new quadratic divergences. The typical magnitude of the SUSY-breaking mass terms added to the SM is

$$m \sim \frac{\langle F \rangle}{M}$$

where $|\langle F \rangle| \sim (\text{mass})^2$ is the magnitude of the SUSY-violating term in the high-energy sector and M is the *messenger scale*, the mass of the particle that transmits the symmetry breaking to the SM fields. The default is mediation by gravity and supergravity interactions; for this, if we would like to obtain $m \sim 1$ TeV,

$$M \sim m_{Pl.} \qquad \langle F \rangle \sim 10^{11}\ \text{GeV}$$

This complete Lagrangian for SM fields, with supersymmetric interactions, SUSY-violating soft terms, and two Higgs doublets, is called the Minimal Supersymmetric Standard Model (MSSM).

We know nothing about the high-energy SUSY-breaking sector, so it would be good not to make restrictive assumptions about this sector in working out the phenomenology of SUSY particles. But the price of this is that our phenomenology should allow the most general values of the soft parameters. There are many of these. A typical parameter set contains 24 parameters,

$$\begin{array}{lll} \mu & \tan\beta & m_A \\ 9 & \hat{q}\ \hat{u}\ \hat{d} & \text{masses.} \\ 6 & \hat{L}\ \hat{e} & \text{masses} \\ 3 & \lambda^a & \text{masses} \end{array}$$

plus 3 cubic scalar couplings for t, b, and τ partners. Often, one sets masses of the first- and second-generation partners equal to minimize the generation of flavor-changing neutral current interactions by SUSY radiative corrections. This gives a 19-parameter model called the pMSSM. The most general model, with arbitrary flavor violation by soft interactions, has over 100 parameters.

The feature of most interest to us in these lectures is that this structure contains a dynamical mechanism for EWSB. To explain this, we must first realize that the assumption of SUSY has changed the problem of EWSB, both for better and for worse. In the SM, we had only one elementary scalar field, which looked anomalous among the many fermion and vector fields. In the MSSM, there is nothing unusual about elementary scalar fields. In fact, we have 17 multiplets of them. On the other hand, it is possible in principle that any of these 17 fiels could acquire vacuum expectation values. For most of these fields, this would be a disaster. For example

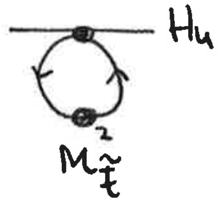
$$\langle \hat{t} \rangle \neq 0$$

breaks color $SU(3)$ and $U(1)$ but not $SU(2)$. Actually, there are regions in the large MSSM parameter space where the $\tilde{t}$ and $\tilde{\tau}$ fields obtain nonzero vacuum values. But, I will shw that there is a large region where it is preferred that it is H_u that obtains a vacuum value.

The key element of this argument is that the soft mass terms for $\tilde{t}$ and $\tilde{\bar{t}}$ generate a $(\text{mass})^2$ for H_u through the top quark Yukawa coupling, which appears in the V_F interaction terms

$$\delta\mathcal{L} = -y_t^2\left(|H_u\tilde{\bar{t}}|^2 + |\tilde{q}\cdot H_u|^2\right)$$

For the $\tilde{\bar{t}}$, for example, this vertex leads to a diagram



$$= (-iy_t^2)\cdot 3\cdot \int \frac{d^4k}{(2\pi)^4}\frac{i}{k^2}(-iM^2_{\tilde{\bar{t}}})\frac{i}{k^2}$$

$$= i\cdot\frac{3y_t^2}{(4\pi)^2}M^2_{\tilde{\bar{t}}}\cdot\log\Lambda^2/M^2_{\tilde{\bar{t}}}$$

The factor 3 comes from color in the loop. The sign of the result corresponds to a *negative* correction to the (mass)^2. If the top quark Yukawa coupling is the most important source of radiative corrections to the Higgs potential, this term can dominate and drive an instability to $\langle H_u\rangle \neq 0$.

We can make this discussion more precise. What we have actually computed is a correction to the renormalization group running of the H_u mass operator. This is part of a system of renormalization group equations for the soft SUSY-breaking parameters. This is a large system of equations, but I would like to highlight the most relevant terms. These are

$$\frac{dM^2_{\tilde{\bar{t}}}}{d\log Q} = \frac{2}{(4\pi)^2}\cdot 1\cdot y_t^2\left[M^2_{\tilde{t}} + M^2_{\tilde{\bar{t}}} + M^2_{H_u} + A_t^2\right] - \frac{8}{3\pi}\alpha_s m^2_{\tilde{g}}$$

$$\frac{dM^2_{\tilde{t}}}{d\log Q} = \frac{2}{(4\pi)^2}\cdot 2\cdot y_t^2\left[M^2_{\tilde{t}} + M^2_{\tilde{\bar{t}}} + M^2_{H_u} + A_t^2\right] - \frac{8}{3\pi}\alpha_s m^2_{\tilde{g}}$$

$$\frac{dM^2_{H_u}}{d\log Q} = \frac{2}{(4\pi)^2}\cdot 3\cdot y_t^2\left[M^2_{\tilde{t}} + M^2_{\tilde{\bar{t}}} + M^2_{H_u} + A_t^2\right]$$

Here $m_{\tilde{g}}$ is the mass of the gluino, the SUSY partner of the gluon, and A_t is the coefficient of a 3-scalar operator involving H_u and the top squarks. The signs are such that the gluino mass contributes a positive correction to the top squark masses as we integrate to smaller Q, as might be expected. The y_t^2 terms generate negative contributions.

The factor 3 in the last equation is a sum over color $SU(3)$ indices, as explained above. In the equation for the mass of $\tilde{\bar{t}}$ there is a factor 2 that comes from a sum over

$SU(2)$ indices in similar loops. It is remarkable that the strongest negative correction is generated for the H_u field. SO if the mass terms for H_u, $\tilde{t}$, $\tilde{\bar{t}}$ are all positive at very short distances and roughly of the same size, the first instability generated as we approach large distances will be that to $\langle H_u \rangle \neq 0$. This is a dynamical explanation for EWSB that includes an explanation why the Higgs field, rather than other scalar fields in the model, should obtain a vacuum expectation value.

The MSSM is thus of great interest as a model of EWSB. It is also interesting for other reasons. The model turns out to repair the difficulty in the SM of matching the observed values of the three SM gauge couplings to the predictions of grand unification. It also contains a number of possible candidates for the particle of cosmic dark matter, including the SUSY partners of the photon, Z, Higgs bosons, and graviton.

There is one more issue that we must discuss. If the SUSY partners of the SM particles do all of these wonderful things, why haven't we discovered them?

The SUSY partners of quarks and gluons have substantial production cross sections at the LHC. Production of SUSY particles is also expected to have a distinctive signature. The quantity

$$R = (-1)^{2J+3B+L}$$

called *R-parity*, is conserved if we have no baryon- or lepton-number violating interactions, as is true in the MSSM. Further, R assigns the quantum number $R = +1$ to all particles of the SM and $R = -1$ to all SUSY partners. So any decay of a SUSY partner must result in the production of another SUSY partner. The lightest SUSY partner must be absolutely stable. In most MSSM parameter sets, the lightest SUSY partner is a neutral particle such as the partner of the photon, Z. This is a heavy particle with weak-interaction-size cross sections on matter; it is expected to exit the LHC detectors without making any signal. Quark and gluon SUSY partners will decay to this particle by emission of quarks and gluons, producing reactions

$$pp \to (2\text{-}4 \text{ jets}) + \not{E}_T$$

where $\not{E}_T$ is unbalanced transverse momentum carried away by unobserved particles recoiling against the visible ones. There has been a dedicated effort to find events of this type at the LHC. A candidate event from ATLAS is shown in Figure 1. However, there are many SM processes that produce this signature, for example,

$$q\bar{q} \to g + g + Z^0 \to \nu\bar{\nu}$$

Cross sections for the production of SUSY particles at the 8 TeV LHC are shown in Figure 2. A SUSY contribution to the jets plus $\not{E}_T$ signature is excluded down to a cross section of about 10 fb, corresponding to squark and gluino masses above 1 TeV.

Most enthusiasts of SUSY, including me, believed that the quark and gluon SUSY partners would have masses in a range around 500 GeV and so should have been discovered at the 7 and 8 TeV LHC. However, there is no rigorous upper limit on the masses of SUSY particles. What we have instead is an argument called "naturalness". If the role of SUSY particles is to cancel the quadratic divergences in the radiative corrections to the Higgs mass, this cancellation should not require extreme fine-tuning. As an example, we might look at the top quark contribution to the 1-loop correction to the Higgs mass

$$\delta\mu^2 = -\frac{3y_t^2}{8\pi^2}\Lambda^2$$

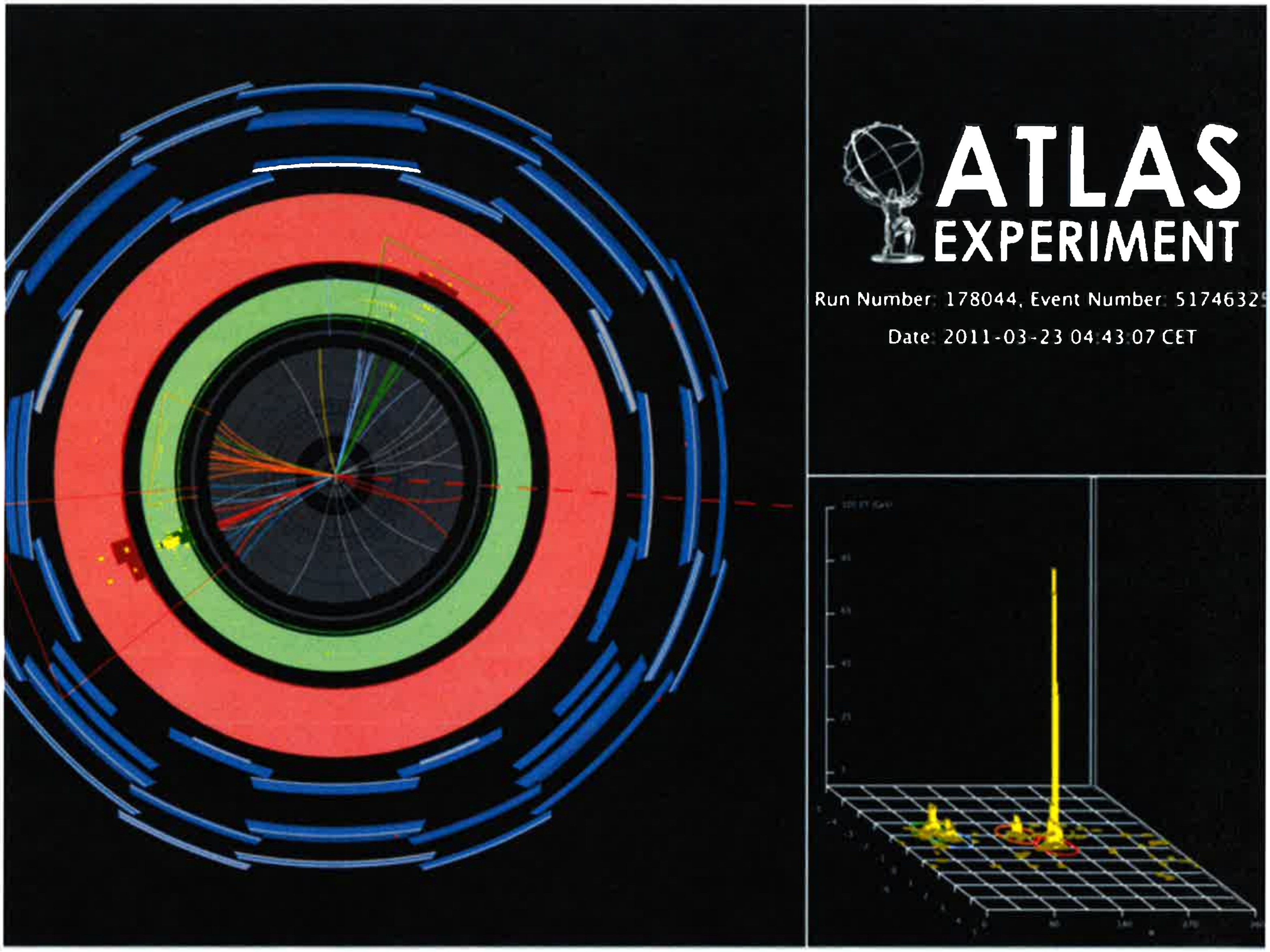
In a SUSY theory that successfully regulates this divergence, we would have

$$\Lambda \sim M_{\tilde{t}} \text{ or } M_{\tilde{t}}$$

This term would then be of the order of

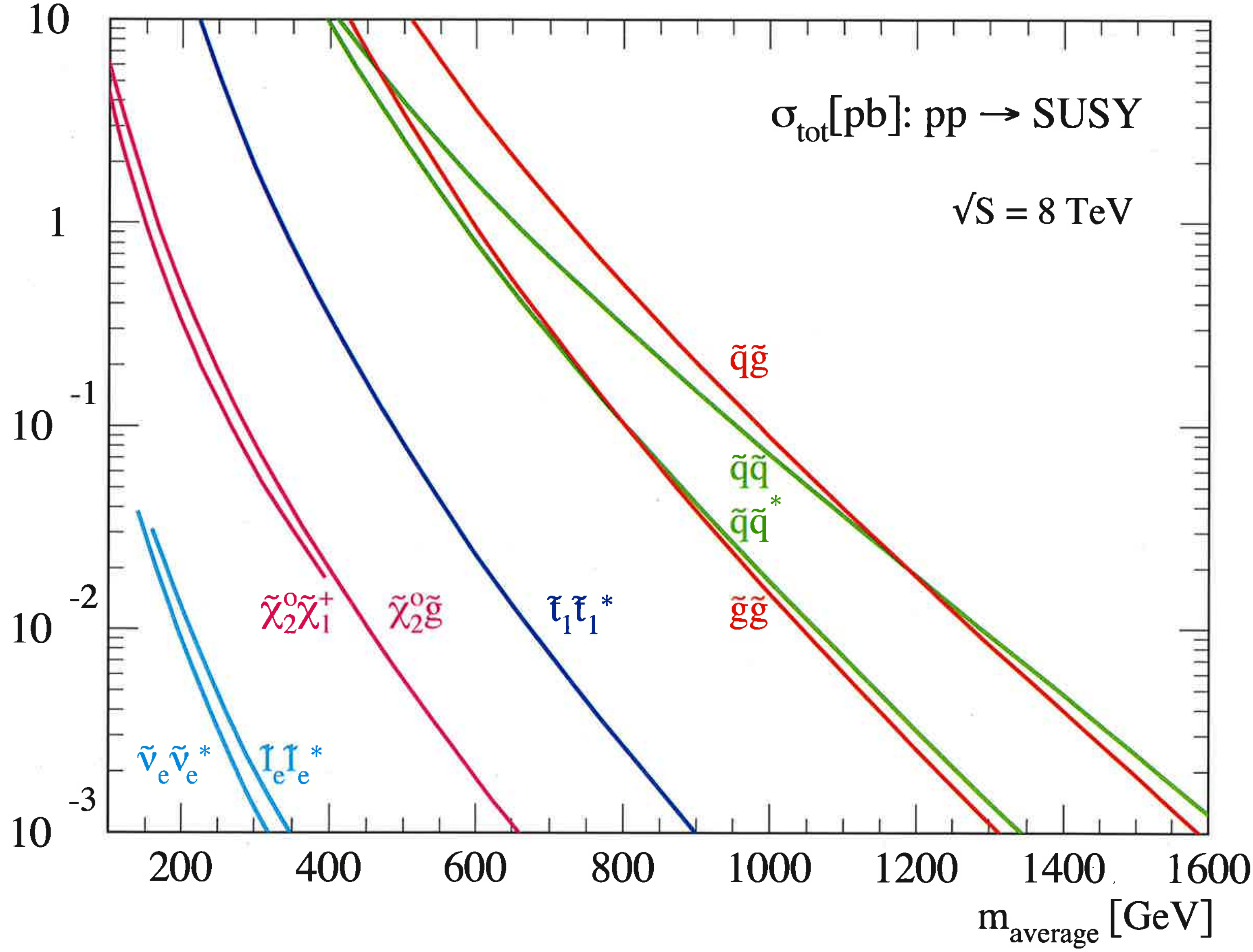
$$\frac{3y_t^2}{8\pi^2} M_{\tilde{t}}^2 \sim \begin{cases} (100 \text{ GeV})^2 & M_{\tilde{t}} = 500 \\ (300 \text{ GeV})^2 & M_{\tilde{t}} = 1500 \text{ GeV} \\ (1000 \text{ GeV})^2 & M_{\tilde{t}} = 5 \text{ TeV} \end{cases}$$

In the first case, only an $\mathcal{O}(1)$ cancellation is required. In the last case, we need this correction to be cancelled in the first two decimal places.



ATLAS
EXPERIMENT
Run Number: 178044, Event Number: 5174632
Date: 2011-03-23 04:43:07 CET

PROSPINO 2


10
1
10 -1
10 -2
10 -3
200
400
600
800
1000
1200
1400
1600
$m_{average}$ [GeV]
σ_{tot}[pb]: pp → SUSY
√S = 8 TeV
$\tilde{q}\tilde{g}$
$\tilde{q}\tilde{q}$
$\tilde{q}\tilde{q}^*$
$\tilde{g}\tilde{g}$
$\tilde{t}_1\tilde{t}_1{}^*$
$\tilde{\chi}^o_2\tilde{\chi}^+_1$
$\tilde{\chi}^o_2\tilde{g}$
$\tilde{\nu}_e\tilde{\nu}_e{}^*$
$\tilde{l}_e\tilde{l}_e{}^*$

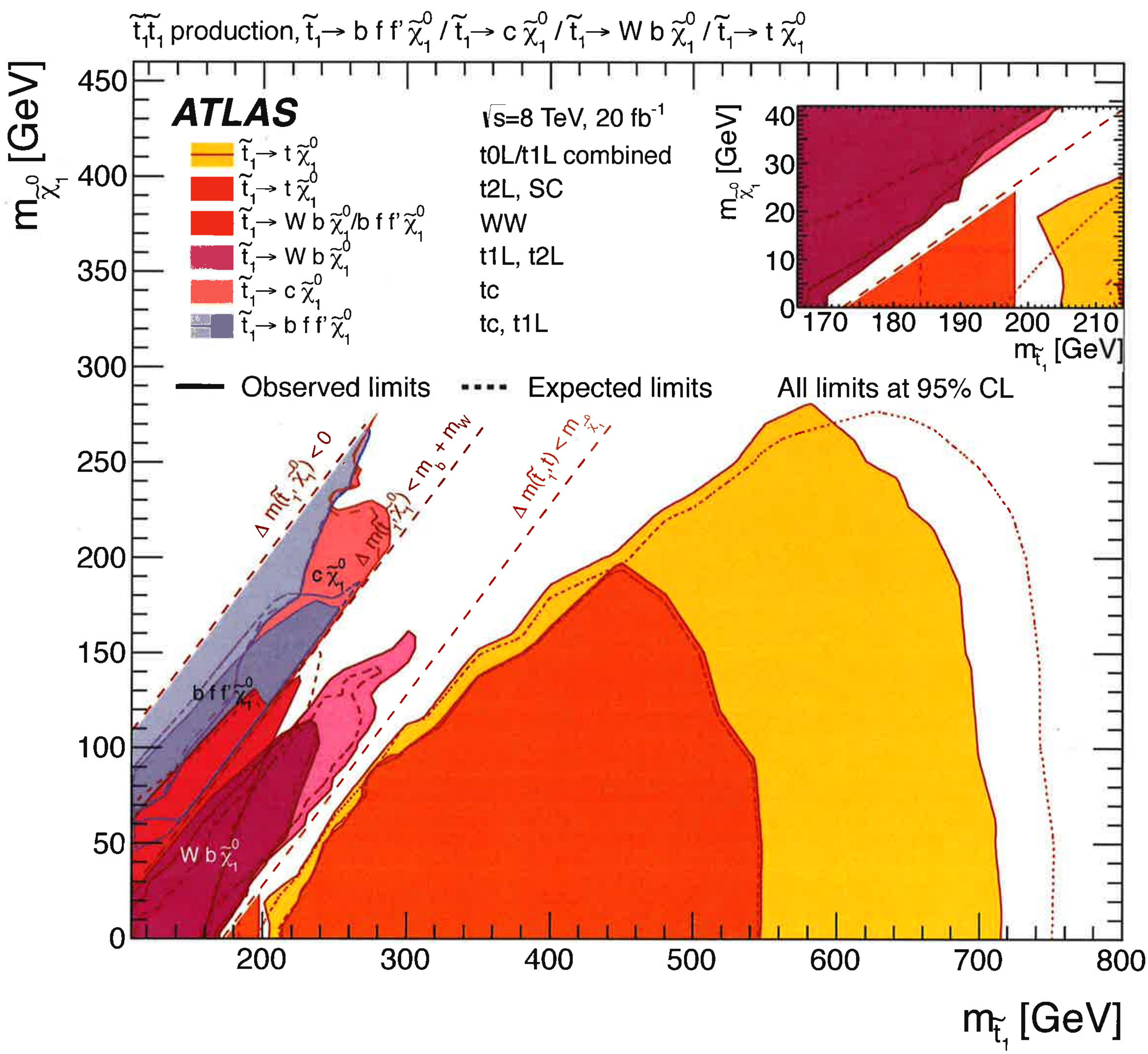
The earliest estimates of the masses of SUSY particles assumed that all of the partners would have approximately equal masses. Many of these analyses were done in a framework called MSUGRA or cMSSM in which the fermion partners and, separately, the gauge boson partners, were taken to have equal masses at a unification scale, before renormalization group corrections. This is a highly restrictive framework, and it is severely constrained by the LHC results.

It was also possible, as first emphasized by Cohen, Kaplan, and Nelson, that the third generation SUSY partners might be lighter than the other fermion partners. Perhaps the only SUSY partners near the TeV scale are those that are required to produce the Higgs potential—the top squarks and the higgsinos. Since the gluino contributes to the top squark masses, it also is restricted in mass in this scenario. The top squarks are very difficult to discover at the LHC. Their cross sections are about a factor of 50 smaller than the cross sections for u and d squarks, and they decay to complex final states hidden below backgrounds from $t\bar{t}$ production. The current ATLAS limits on direct top squark production are shown in Figure 3. The mass region below 600 GeV is largely excluded, but the LHC constraints end well below 1 TeV.

Ther higgsinos are even more difficult to discover at the LHC. They are produced only by electroweak interactions, so their cross sections are small, and the $\tilde{h}^+$ and $\tilde{h}^0$ states are expected to be almost degenerate, leading to very little visible energy in the event. The LHC has not significantly improved on the limit from LEP2 of about 100 GeV in the higgsino masses.

Probably the best bet for a SUSY discovery at the LHC is in the search for gluino pair production, with decay through real or virtual top squarks to b quark jets plus $\not{E}_T$. The current LHC limit is abot 1.3 TeV. This will be pushed close to 2 TeV in the next few years—or, the particle will be discovered.

Thus, SUSY is challenged experimentally but not yet excluded. It remains an extremely attracting theory in many respects, including its theory of EWSB. I am excited to see what the next run of the LHC will bring.



ATLAS limits on $pp \to \tilde{t}\,\tilde{t}$